

# Less space for a new family of fermions

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## Abstract

We investigate the experimentally allowed parameter space of an extension of the standard model (SM3) by one additional family of fermions. Therefore we extend our previous study of the CKM like mixing constraints of a fourth generation of quarks. In addition to the bounds from tree-level determinations of the  $3 \times 3$  CKM elements and FCNC processes ( $K$ -,  $D$ -,  $B_d$ -,  $B_s$ -mixing and the decay  $b \rightarrow s\gamma$ ) we also investigate the electroweak  $S$ ,  $T$ ,  $U$  parameters, the angle  $\gamma$  of the unitarity triangle and the rare decay  $B_s \rightarrow \mu^+\mu^-$ . Moreover we improve our treatment of the QCD corrections compared to our previous analysis. We also take leptonic contributions into account, but we neglect the mixing among leptons. As a result we find that typically small mixing with the fourth family is favored, but still some sizeable deviations from the SM3 results are not yet excluded. The minimal possible value of  $V_{tb}$  is 0.93. Also very large CP-violating effects in  $B_s$  mixing seem to be impossible within an extension of the SM3 that consists of an additional fermion family alone. We find a delicate interplay of electroweak and flavor observables, which strongly suggests that a separate treatment of the two sectors is not feasible. In particular we show that the inclusion of the full CKM dependence of the  $S$  and  $T$  parameters in principle allows the existence of a degenerate fourth generation of quarks.

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# 1 Introduction

Increasing the number of fermion generations (see [1] for a review and [2] for an update) is probably the most obvious extension of the usual standard model with three generations (SM3). Although being popular in the 80s, such a possibility was discarded for a long time. Recently these models (SM4) celebrated a kind of resurrection. Partly, this was due to the fact that a fourth generation is not necessarily in conflict with electroweak precision observables [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

Besides being a straightforward extension of the SM3, an increase of the number of fermion generations leads also to several desired effects:

- The authors of [7, 9, 11, 12, 13, 15] have shown that a fourth generation softens the current low Higgs mass bounds from electroweak precision observables, see e.g. [16], by allowing considerably higher values for the Higgs mass.
- It might solve problems related to baryogenesis: An additional particle family could lead to a sizeable increase of the measure of CP-violation, see [17, 18]. Moreover, such an extension of the SM would increase the strength of the phase transition, see [19, 20, 21].
- The gauge couplings can in principle be unified without invoking SUSY [22].
- New heavy fermions lead to new interesting effects due to their large Yukawa couplings, see e.g [23, 24]. Moreover dynamical electroweak symmetry breaking might be triggered by these heavy new fermions [25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. This mechanism can also be incorporated in models with warped extra dimensions, as done in [36, 37].

There are also some modest experimental deviations that could be explained by the existence of a fourth generation:

- A new family might cure certain problems in flavor physics (CP-violation in  $B_s$ -mixing,  $K - \pi$ -puzzle,  $\epsilon_k$  anomaly,...) see e.g. [38, 39, 40, 41, 42] for some recent work and e.g. [43, 44] for some early work on 4th generation effects on flavor physics.
- Investigations of lepton universality show a value of the PMNS element  $V_{e4} \neq 0$  at the  $2.5 \sigma$  level [45].

For more arguments in favor of a fourth generation see e.g. [2] and also [46, 47, 48, 49]. We conclude the list by repeating our statement from [50]: *In view of the (re)start of the LHC, it is important not to exclude any possibility for new physics scenarios simply due to prejudices.* Direct search strategies for heavy quarks at the LHC are worked out e.g. in [31, 51, 52, 53, 54, 55, 56, 57]. Signatures and consequences for collider physics, such as the modification of production rates, have been studied e.g. in [58, 31, 59, 60, 61].

In this work we extend our analysis in [50], where we performed an exploratory study of the allowed parameter range for the CKM like mixing of hypothetical quarks of a fourth generation. Adding one generation of quarks results in several new parameters. In particular, we have the new masses  $m_{b'}$  and  $m_{t'}$ , and nine parameters (six angles and three phases) in the  $4 \times 4$  CKM matrix (compared to three angles and one phase in the SM3). Following our previous strategy, we consecutively add bounds on the CKM structure of the SM4 and perform a scan through the parameter space of the model to identify the allowed regions;

while this treatment is insufficient to fit for the central values or standard deviation of the model parameters, it gives a very reasonable idea of the experimentally possible parameter space allowing for statements on the size of effects of the model on particularly interesting flavor observables. Apart from the unitarity of the  $4 \times 4$  matrix and the direct bounds on the quark masses, the most important input comes from direct measurements of the absolute values of CKM matrix elements, e.g. from  $\beta$  decay. In [50] the next step was the inclusion of flavor observables sensitive to FCNC, mediated i.e. by box or penguin diagrams. This led to some surprising results regarding the possible size of the quark mixing with the fourth generation quark, as rather large values for the mixing angle  $s_{34}$  could not be excluded.

However, in [62] Chanowitz found that the parameter sets, which we gave as an example for large mixing with the fourth generation, are excluded by electroweak precision constraints, in particular by the oblique corrections [63]. Moreover, Chanowitz performed the whole electroweak fit for four different values of the mass of the  $t'$  quark. Here, some assumptions were used: i) the lepton masses are fixed to  $m_{l4} = 145$  GeV and  $m_{\nu_4} = 100$  GeV, ii) lepton mixing is not included, iii) the mass difference of the heavy quark doublet is fixed to  $m_{t'} - m_{b'} = 55$  GeV, iv) only mixing between the third and fourth family was included. Assumptions iii) and iv) were also tested in [62].

Therefore, we supplement the analysis of the flavor sector by the  $S$ ,  $T$  and  $U$  parameters; also the lepton masses of the fourth generation have to be taken into account. For the present work we assume that the neutrinos have Dirac character and neglect the possible mixing of the fourth neutrino in the lepton sector. Moreover, we extend the set of our FCNC observables to include also  $B_s \rightarrow \mu^+ \mu^-$  and we improve the simplified treatment of the decay  $b \rightarrow s \gamma$  by using the full leading logarithmic result. Concerning the tree-level determination of the CKM elements we include now also the experimental results for the angle  $\gamma$  of the unitarity triangle, which gives a direct constraint on CKM phases. Similar studies have been recently performed e.g. in [64, 65, 66, 67]

In Section 2 we present all experimental constraints we use in our analysis. We start with the parameterization of  $V_{CKM4}$  in Section 2.1, next we discuss briefly tree-level determinations of CKM elements and direct mass limits. The electroweak parameters  $S$ ,  $T$  and  $U$  will be investigated before reviewing the FCNC constraints. We end Section 2 with the allowed regions for deviations of the SM4 results from the SM3 values.

In Section 3 we determine the bounds on the parameters of the model. After explaining our general strategy in 3.1, we present the results for the different mixing angles of  $V_{CKM4}$ , the new results for  $V_{cx}$  and  $V_{tx}$  ( $x = d, s, b$ ), and allowed effects of a fourth generation in neutral meson mixing.

In Section 4 we give a Wolfenstein-like expansion of the  $4 \times 4$  CKM matrix. With the additional information from the electroweak sector, tighter constraints on the fourth generation quark mixing can be utilized leading to a simplified expansion.

We conclude with Section 5.

## 2 Constraints on $V_{CKM4}$

### 2.1 Parameterization of $V_{CKM4}$

In the SM3 the mixing between quarks is described by the unitary  $3 \times 3$  CKM matrix [68, 69], which can be parameterized by three angles,  $\theta_{12}$ ,  $\theta_{13}$  and  $\theta_{23}$  ( $\theta_{ij}$  describes the strength of

the mixing between the  $i$ th and  $j$ th family) and the CP-violating phase  $\delta_{13}$ . The so-called standard parameterization of  $V_{CKM3}$  reads

$$V_{CKM3} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \quad (2.1)$$

with

$$s_{ij} := \sin(\theta_{ij}) \quad \text{and} \quad c_{ij} := \cos(\theta_{ij}). \quad (2.2)$$

Extending the minimal standard model to include a fourth family of fermions (SM4) introduces 3 additional angles in the CKM matrix  $\theta_{14}, \theta_{24}$  and  $\theta_{34}$  and 2 additional CP-violating phases  $\delta_{14}$  and  $\delta_{24}$ . To determine the allowed range for these new parameters we use an exact parameterization of the  $4 \times 4$  CKM matrix. We have chosen the one suggested by Botella and Chau [70]<sup>1</sup>, Fritzsch and Plankl [71]<sup>2</sup> and also by Harari and Leurer [72].

$$V_{CKM4} = \begin{pmatrix} c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ -c_{23}c_{24}s_{12} - c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} & c_{12}c_{23}c_{24} - c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} & c_{13}c_{24}s_{23} & c_{14}s_{24}e^{-i\delta_{24}} \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} & -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} & -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})} & \\ -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} + c_{34}s_{12}s_{23} & -c_{12}c_{34}s_{23} - c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} & c_{13}c_{23}c_{34} & c_{14}c_{24}s_{34} \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} & -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} & -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} & \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} & -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} & -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} & \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} & +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} & & \\ -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} & -c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}} + c_{12}s_{23}s_{34} & -c_{13}c_{23}s_{34} & c_{14}c_{24}c_{34} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} & -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} & -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} & \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}} - s_{12}s_{23}s_{34} & +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} & -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})} & \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} & +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} & & \end{pmatrix} \quad (2.3)$$

For our strategy the explicit form of  $V_{CKM4}$  does not matter, it is only important that the parameterization is exact. Besides the nine parameters of  $V_{CKM4}$  we have also the masses of the fourth generation particles, which we denote as  $m_{b'}$ ,  $m_{t'}$ ,  $m_{l_4}$  and  $m_{\nu_4}$ . We do not include leptonic mixing, yet.

## 2.2 Experimental bounds

In this section we summarize the experimental constraints that have to be fulfilled by the parameters of the fourth family.

The elements of the  $3 \times 3$  CKM matrix have been studied intensely for many years and precision data on most of them is available. In principle there are two different ways to determine the CKM elements. On the one hand, they enter charged weak decays already at tree-level and a measurement of e.g. the corresponding decay rate provides direct information on the CKM elements (see e.g. [73] and references therein). We will refer to such constraints as *tree-level constraints*. On the other hand, processes involving flavor-changing neutral

<sup>1</sup>In the published paper of Botella and Chau there is a typo in the element  $V_{td}$ : in the last term of  $V_{td}$  the factor  $s_y$  has to be replaced by  $c_y$ .

<sup>2</sup>In the published paper of Fritzsch and Plankl there is a typo in the element  $V_{cb}$ : the factor  $c_{23}$  has to be replaced by the factor  $s_{23}$ .

currents (FCNC) are forbidden at tree-level and only come into play at loop level via the renowned Penguin and Box diagrams. These processes provide strong bounds — referred to as *FCNC constraints* — on the structure of the CKM matrix and its elements as well as on the masses of the heavy virtual particles appearing in the loops.

We will start with the tree-level constraints, since they only depend on the CKM elements and not on the fermion masses. Next we consider mass constraints on the fourth family members from direct searches at colliders. Since the oblique electroweak parameters are expected to reduce the allowed range of masses for a new fermion family notably, we consider them next and finally we discuss the FCNC constraints.

### 2.2.1 Tree-level constraints for the CKM parameters

Since the (absolute) value of only one CKM element enters the theoretical predictions for weak tree-level decays, no GIM mechanism or unitarity condition has to be assumed. By matching theory and experiment the matrix element can be extracted *independently* of the number of generations <sup>3</sup>. Therefore, all tree-level constraints have the same impact on the  $4 \times 4$  matrix as they have on the  $3 \times 3$  one.

We take the PDG values [74] for our analysis:

	absolute value	relative error	direct measurement from
$V_{ud}$	$0.97418 \pm 0.00027$	0.028%	nuclear beta decay
$V_{us}$	$0.2255 \pm 0.0019$	0.84%	semi-leptonic K-decay
$V_{ub}$	$0.00393 \pm 0.00036$	9.2%	semi-leptonic B-decay
$V_{cd}$	$0.230 \pm 0.011$	4.8%	semi-leptonic D-decay
$V_{cs}$	$1.04 \pm 0.06$	5.8%	(semi-)leptonic D-decay
$V_{cb}$	$0.0412 \pm 0.0011$	2.7%	semi-leptonic B-decay
$V_{tb}$	$> 0.74$		(single) top-production

In the following, we denote the absolute values in the table above as  $|V_i| \pm \Delta V_i$ . In addition to the above tree-level constraints there exists a direct bound on the CKM angle

$$\gamma = \arg \left( -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right) .$$

It can be extracted via the decays  $B \rightarrow DK, D\pi$  [75, 76, 77, 78]. In principle the extraction of  $\gamma$  might be affected by the presence of a fourth generation of fermions [79], but it was shown in [80] that these effects are negligible. Therefore,  $\gamma$  gives direct information on the phases of the CKM matrix; with three CP violating phases present, this can provide a useful piece of information. We use the CKMfitter value from [81] (update of [82, 83])

$$\gamma = 73^{\circ-25^{\circ}}_{+22^{\circ}} \pm 2^{\circ} , \quad (2.4)$$

where the last error accounts for the tiny additional uncertainty due to the additional fermion generation.

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<sup>3</sup>There is, however, one loop hole: In [45] the possibility of lepton mixing reducing the accuracy of the determination of e.g.  $V_{ud}$  was discussed. Since we use the more conservative error estimate from the PDG, our relative error is similar to the final error of Lackner and Menzel, who started with a more ambitious error for  $V_{ud}$  in their analysis [45].

### 2.2.2 Direct mass limits for the fourth family

The PDG [74] gives from direct searches the following mass limits for a fourth family

$$m_{\nu_4} > 80.5...101.5 \text{ GeV} , \quad (2.5)$$

$$m_{l_4} > 100.8 \text{ GeV} , \quad (2.6)$$

$$m_{b'} > 128...268 \text{ GeV} , \quad (2.7)$$

$$m_{t'} > 256 \text{ GeV} . \quad (2.8)$$

The mass bound on the heavy neutrino depends on the type of neutrino (Dirac or Majorana) and whether one considers a coupling of the heavy neutrino to  $e^-$ ,  $\mu^-$  or  $\tau^-$ . It is interesting to note that *LEP results in combination with [84] exclude a fourth stable neutrino with  $m < 2400 \text{ GeV}$*  [74]. The quark mass bounds are obtained from direct searches at TeVatron [85, 86], which were recently updated [87, 88]

$$m_{b'} > 338 \text{ GeV}, \quad m_{t'} > 335 \text{ GeV}. \quad (2.9)$$

In [89] it was pointed out that in deriving these bounds assumptions about the couplings of the fourth generation have been made (in [87] it is e.g. explicitly assumed that the  $b'$  is shortlived and that it decays exclusively to  $tW^-$ , which corresponds to demanding  $V_{ub'} \approx 0 \approx V_{cb'}$ ,  $m_{b'} < m_{t'}$  and  $V_{tb'}$  is not extremely small). Without these assumptions the mass bounds can be weaker, as the extraction of the masses has to be combined with the extraction of the CKM couplings. The inclusion of this dependence is beyond the scope of the current work. For some recent papers concerning the mass extraction of leptons and quarks, see [90, 91].

In this work we investigate heavy quark masses in the range of 280 GeV to 650 GeV heavy charged lepton masses in the range of 100 GeV to 650 GeV and heavy neutrino masses in the range of 90 GeV to 650 GeV. Note that the triviality bound from unitarity of the  $t't'$   $S$ -wave scattering [92] indicates a maximal  $t'$  mass of around 504 GeV [93]. However, this estimate is based on tree-level expressions and while it seems prudent to treat too high quark masses with a grain of salt, one should not disregard higher masses based on this estimate alone. In this context it would be desirable to have e.g. a lattice study of the effect of very heavy (fourth generation) quarks.

### 2.2.3 Electroweak constraints

We present here the expressions for the oblique electroweak  $S, T$  and  $U$  parameters [63] in the presence of a fourth generation. They were originally defined as

$$\alpha S = 4e^2 \frac{d}{dq^2} [\Pi_{33}(q^2) - \Pi_{3Q}(q^2)] \Big|_{q^2=0} , \quad (2.10)$$

$$\alpha T = \frac{e^2}{x_W \bar{x}_W M_Z^2} [\Pi_{11}(0) - \Pi_{33}(0)] , \quad (2.11)$$

$$\alpha U = 4e^2 \frac{d}{dq^2} [\Pi_{11}(q^2) - \Pi_{33}(q^2)] \Big|_{q^2=0} , \quad (2.12)$$

with the electric coupling  $\alpha$  and  $e$ ,  $\Pi_{xy}$  denotes the virtual self-energy contributions to the weak gauge bosons and with the Weinberg angle expressed as  $x_W = \sin^2 \theta_W$  and  $\bar{x}_W = 1 - x_W$ . In the first paper of Ref. [63]  $\bar{x}_W M_Z^2$  was approximated by  $M_W^2$ . The  $T$  parameter is related to the famous  $\rho$ -parameter [94, 95, 92]

$$\rho := \frac{M_W^2}{\bar{x}_W M_Z^2} =: 1 + \Delta\rho \quad (2.13)$$

$$= 1 + \alpha T. \quad (2.14)$$

In practice, it turns out to be considerably simpler to reexpress the derivatives in  $S$  and  $U$  as differences

$$\left. \frac{d}{dq^2} \Pi_{XY}(q^2) \right|_{q^2=0} \approx \frac{\Pi_{XY}(M_Z^2) - \Pi_{XY}(0)}{M_Z^2}. \quad (2.15)$$

This approximation works very well for  $m_{new} \gg M_Z$  and it is used by the PDG [74]. We will use however the original definitions given in Eqs. (2.10), (2.11), (2.12) with  $M_W^2 = \bar{x}_w M_Z^2$ , because there are no correction terms and our expressions are exact.

Next only the new physics contributions to the  $S$ ,  $T$  and  $U$  parameters will be considered, as the SM values of the oblique parameters are by definition set to zero. Fit results for the allowed regions of the  $S$ ,  $T$  and  $U$  parameters are obtained e.g. by the PDG [74], EWWG [96], Gfitter [16] and most recent in [14]. Note that the more recent analyses [16, 14] differ significantly from the old (November 2007) PDG version. Due to more refined experimental results and an improved theoretical understanding the best fit values shifted significantly towards higher values of  $S$  and  $T$ , see Fig. 1 for the Gfitter S-T ellipse [97]; this somewhat relaxes the previously observed tension with an additional fermion generation.

In the presence of a fourth generation the fermionic contribution to these parameters (before the necessary subtraction of the SM contribution) reads

$$S = \frac{N_c}{6\pi} \sum_{f=1}^4 \left[ 1 - \frac{1}{3} \ln \frac{m_{u_f}^2}{m_{d_f}^2} \right] + \frac{1}{6\pi} \sum_{f=1}^4 \left[ 1 + \ln \frac{m_{\nu_f}^2}{m_{l_f}^2} \right], \quad (2.16)$$

$$T = \frac{N_c}{16\pi x_W \bar{x}_W M_Z^2} \left[ \sum_{q=u,d,s,\dots,t',b'} m_q^2 - \sum_{f=1}^4 \sum_{f'=1}^4 |V_{u_f d_{f'}}|^2 F_T(m_{u_f}^2, m_{d_{f'}}^2) \right] \quad (2.17)$$

$$+ \frac{1}{16\pi x_W \bar{x}_W M_Z^2} \left[ \sum_{l=\nu_e, e^-, \dots, \nu_4, l_4^-} m_l^2 - \sum_{f=1}^4 \sum_{f'=1}^4 |V_{\nu_f l_{f'}}|^2 F_T(m_{\nu_f}^2, m_{l_{f'}}^2) \right], \quad (2.18)$$

$$U = \frac{N_c}{3\pi} \left[ \sum_{f=1}^4 \sum_{f'=1}^4 |V_{u_f d_{f'}}|^2 F_U(m_{u_f}^2, m_{d_{f'}}^2) - \frac{5}{6} \sum_{f=1}^4 1 \right] \\ + \frac{1}{3\pi} \left[ \sum_{f=1}^4 \sum_{f'=1}^4 |V_{\nu_f l_{f'}}|^2 F_U(m_{\nu_f}^2, m_{l_{f'}}^2) - \frac{5}{6} \sum_{f=1}^4 1 \right]. \quad (2.19)$$

$u_f$  denotes the up-type quark of the  $f$ th generation,  $d_f$  the down-type quark of the  $f$ th generation,  $l_f$  the charged lepton of the  $f$ th generation and  $\nu_f$  the neutrino of the  $f$ th



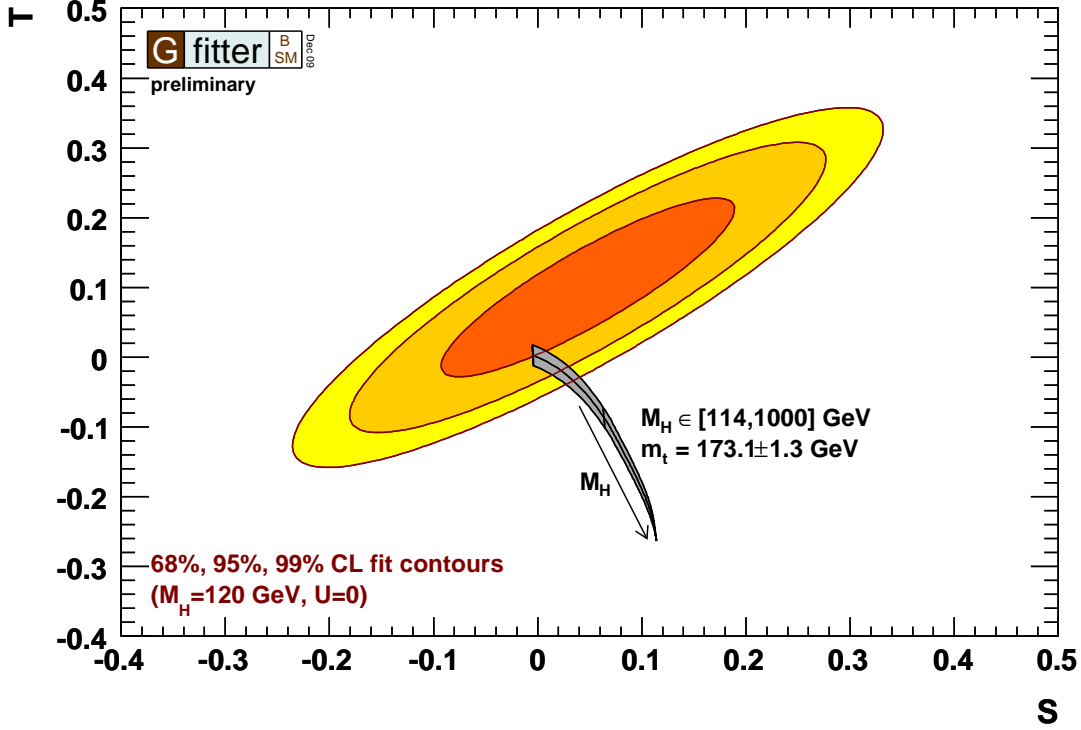


Figure 1: Fit of the electroweak oblique parameters  $S$  and  $T$ . The plot is taken from [97].

generation. We have used the following functions

$$F_T(m_1^2, m_2^2) := 2 \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2}, \quad (2.20)$$

$$F_U(m_1^2, m_2^2) := 2 \frac{m_1^2 m_2^2}{(m_1^2 - m_2^2)^2} + \left( \frac{m_1^2 + m_2^2}{2(m_1^2 - m_2^2)} - \frac{m_1^2 m_2^2 (m_1^2 + m_2^2)}{(m_1^2 - m_2^2)^3} \right) \ln \frac{m_1^2}{m_2^2}. \quad (2.21)$$

Both functions are symmetric in their arguments.

The formula for  $S$  is very well known - see e.g. [63, 5, 11, 62]. Using instead the PDG definition we would obtain the following corrections terms to  $S$  for heavy quark masses ( $m_q^2 \gg M_Z^2$ )

$$S_q^{corr.} = \frac{N_c}{6\pi} \left[ \frac{M_Z^2}{3m_{b'}^2} \left( -\frac{1}{2} + \frac{2}{3}x_W - \frac{4}{9}x_W^2 \right) + \frac{M_Z^2}{3m_{t'}^2} \left( -\frac{1}{2} + \frac{4}{3}x_W - \frac{16}{9}x_W^2 \right) \right], \quad (2.22)$$

$$S_l^{corr.} = -\frac{1}{6\pi} \left[ \frac{M_Z^2}{6m_{\nu_4}^2} + \frac{M_Z^2}{2m_{l_4}^2} \left( -\frac{1}{3} + \frac{1}{3}x_W - 3x_W^2 \right) \right], \quad (2.23)$$

which are very small for the allowed mass ranges of the fourth family members. In the parameter  $T$  no mixing was usually assumed. We give here the full CKM and PMNS dependence. Our expression for  $T$  in Eq. (2.18) agrees with the one quoted in [62], if we make the same assumptions (only 4 – 3 mixing or 4 – 3 and 4 – 2 mixing is considered).

By defining the SM3 values for  $S$  and  $T$  as zero, we only need to take the additional contributions due to the fourth generation into account. Keeping the full, previously neglected



CKM dependences, we obtain

$$S_4 = \frac{1}{3\pi} \left[ 2 + \ln \frac{m_{b'} m_{\nu_4}}{m_{t'} m_{l_4}} \right], \quad (2.24)$$

$$T_4 = \frac{N_c}{16\pi x_w \bar{x}_w M_Z^2} \left[ m_{b'}^2 + m_{t'}^2 - \sum_{f=1}^4 \sum_{f'=1}^4 |V_{u_f d_{f'}}|^2 F_T(m_{u_f}^2, m_{d_{f'}}^2) + F_T(m_t^2, m_b^2) \right] \\ + \frac{1}{16\pi x_w \bar{x}_w M_Z^2} \left[ m_{l_4}^2 + m_{\nu_4}^2 - \sum_{f=1}^4 \sum_{f'=1}^4 |V_{\nu_f l_{f'}}|^2 F_T(m_{\nu_f}^2, m_{l_{f'}}^2) \right], \quad (2.25)$$

$$U_4 = -\frac{N_c}{6\pi} \left[ |V_{t'd}|^2 \ln \frac{m_{t'}^2}{m_d^2} + |V_{t's}|^2 \ln \frac{m_{t'}^2}{m_s^2} + |V_{t'b}|^2 \ln \frac{m_{t'}^2}{m_b^2} + |V_{ub'}|^2 \ln \frac{m_{b'}^2}{m_u^2} + |V_{cb'}|^2 \ln \frac{m_{b'}^2}{m_c^2} \right. \\ \left. - 2|V_{tb'}|^2 F_U(m_t^2, m_{b'}^2) - 2|V_{t'b'}|^2 F_U(m_{t'}^2, m_b^2) \right] \\ - \frac{1}{6\pi} \left[ |V_{\nu_4 e}|^2 \ln \frac{m_{\nu_4}^2}{m_e^2} + |V_{\nu_4 \mu}|^2 \ln \frac{m_{\nu_4}^2}{m_\mu^2} + |V_{\nu_4 \tau}|^2 \ln \frac{m_{\nu_4}^2}{m_\tau^2} + |V_{\nu_e l_4}|^2 \ln \frac{m_{l_4}^2}{m_{\nu_e}^2} + |V_{\nu_\mu l_4}|^2 \ln \frac{m_{l_4}^2}{m_{\nu_\mu}^2} \right. \\ \left. + |V_{\mu\tau l_4}|^2 \ln \frac{m_{l_4}^2}{m_{\mu\tau}^2} - 2|V_{\nu_4 l_4}|^2 F_U(m_{\nu_4}^2, m_{l_4}^2) \right] - \frac{10}{9\pi} + U_{SM3}. \quad (2.26)$$

$S_4$  has a large positive contribution of about 0.21 which is independent of the parameters (masses and mixing) of the model. This value can, however, be diminished by the second logarithmic term that depends on the fermion masses.

In the SM3 the only significant contribution to  $T$  reads

$$T_4 = \frac{N_c}{16\pi x_w \bar{x}_w M_Z^2} [m_t^2 - F_T(m_t^2, m_b^2)]. \quad (2.27)$$

Here safely  $V_{tb} = 1$  can be assumed. In the SM4, however,  $V_{tb}$  can in principle differ significantly from one, therefore we have the correction term in “ $+F_T(m_t^2, m_b^2)$ ” in the formula for  $T_4$ . We also have included all previously neglected mixing terms within the SM3 particles. In principle we also should correct for the charm-strange contribution and for the up-down contribution with “ $+F_T(m_u^2, m_d^2) + F_T(m_c^2, m_s^2)$ ”, but their numerical effect is considerably below one per mille of the top-bottom contribution, so we do not show these two additional correction terms in the formula for  $T_4$ .

With the help of the  $S$  and  $T$  parameter Chanowitz [62] could exclude the three parameter sets, which we gave in [50] as an example for a very large mixing between the third and the fourth generation; these sets have passed all bounds set by precision flavor observables. We confirm the numbers from table I in [62]. We also tested the approximation of taking only 3 – 4 mixing into account: Comparing with the full CKM dependence the differences are below 6 % for these three parameter sets.

To simplify the expression for  $U$  we approximated

$$F_U(m_1^2, m_2^2) \approx -\frac{1}{2} \ln \frac{m_1^2}{m_2^2} \quad \text{for } m_1^2 \ll m_2^2. \quad (2.28)$$

Moreover we have only shown the contributions of the 4th family explicitly in Eq. (2.26), the previously neglected rest is denoted by  $U_{SM3}$ . It will be interesting to see in a future analysis, whether the large logarithms in the lepton sector will lead to strong constraints on the PMNS-matrix. In the literature it is typically assumed that  $U$  is very small, see [5] for a notable exception. To our knowledge we incorporate for the first time the full CKM dependence in  $U$ . We find that arbitrary values for mixing and mass parameters could in principle generate values as large as 7.5 for  $U^4$ . If one only takes into account mixing parameters that pass the tree-level flavor constraints still values of  $\mathcal{O}(0.1)$  seem to be possible; however, in this case we observe a simultaneous blow up of the  $T$  parameter. For  $T < 0.4$ ,  $U$  does not exceed 0.06. Note that, while still small, this value is larger than the 0.02 effect expected without flavor mixing [11].

At this point a few comments are appropriate: first, we would like to point out that our implementation of the  $S$  and  $T$  parameter is not “exact” from the SM4 point of view. In principle one would have to perform a full reanalysis of all electroweak data from the SM4 perspective to fit the new values of  $S$  and  $T$ , as advocated for in [62]. This is, of course, beyond the scope of the present work and it is generally accepted that a large deviation of the oblique parameters from their SM values cannot be accommodated in models that do not introduce new particles coupling to fermions [74]. Secondly, we will henceforth neglect the effect of lepton mixing due to a non-trivial modification of the PMNS matrix — the off diagonal elements including the fourth neutrino are in any case required to be small, see [45].

As a first step in our analysis, we only take the tree-level constraints on  $V_{CKM4}$  into account and investigate the parameter ranges that pass the  $S - T$  test at the 95% confidence level<sup>5</sup>. The following values for the SM fit of the oblique parameters are used [14]<sup>6</sup>

$$S_{\text{bestfit}} = 0.03, \quad \sigma_S = 0.09, \quad (2.29)$$

$$T_{\text{bestfit}} = 0.07, \quad \sigma_T = 0.08, \quad (2.30)$$

$$\rho_{\text{corr}} = 0.867, \quad (2.31)$$

where  $\sigma_x$  gives the standard deviation of  $x$ . The  $S$  and  $T$  parameters are not independent quantities; the strength of this correlation is given by  $\rho_{\text{corr}}$ .

As our work focuses on the flavor aspects of the 4th generation scenario, a short comment on the famous S-T-ellipses seems to be in order. If the  $U$  parameter stays close to zero for some physics model it is feasible to set  $U = 0$  and work with  $S$  and  $T$  alone. In this case the probability distribution of  $S$  and  $T$  reduces to a two-dimensional Gaussian distribution in the S-T plane centered on the best fit values. Due to the different  $\sigma_x$  and due to the strong correlation the distribution is essentially squeezed and rotated. Hence, the “equiprobability” lines are no longer circles but ellipses. In fact the two dimensional case is somewhat special as the problem of finding the ellipse encircling an area corresponding to certain probability  $P$  can be solved analytically. The equation determining the contour for a given confidence

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<sup>4</sup>We did not check whether this is the largest possible value.

<sup>5</sup>For the final investigation of the allowed parameter range of a fourth fermion family we will use the 99% confidence level.

<sup>6</sup>Note that this fit is the most restrictive one currently available. Using instead the results from Gfitter a little more space is left for a fourth family. We simply decided to use the most recent numbers.

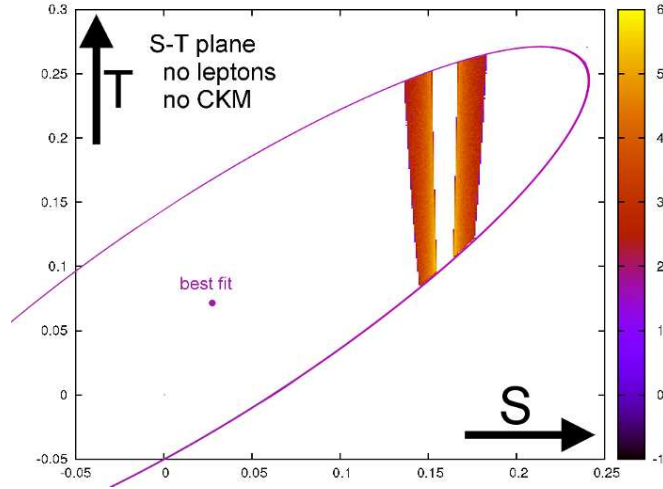
level  $CL$  is then given by

$$\begin{pmatrix} S - S_{\text{best fit}} \\ T - T_{\text{best fit}} \end{pmatrix}^T \begin{pmatrix} \sigma_S \sigma_S & \sigma_S \sigma_T \rho \\ \sigma_S \sigma_T \rho & \sigma_T \sigma_T \end{pmatrix}^{-1} \begin{pmatrix} S - S_{\text{best fit}} \\ T - T_{\text{best fit}} \end{pmatrix} = -2 \ln(1 - CL) . \quad (2.32)$$

Already at that stage we find some interesting results:

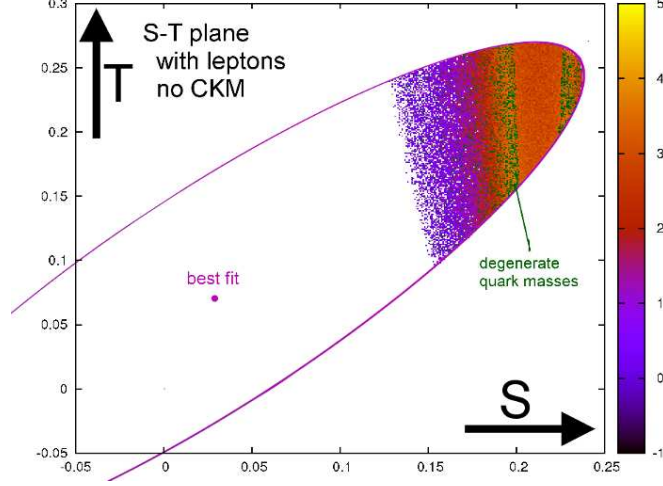
1.  $S - T$  test with ‘no leptons’ + ‘no  $V_{CKM}$ ’

We do not take into account leptons as well as mixing of the quarks. By neglecting the leptonic contribution one can, of course, not make any conclusions as to how restrictive the oblique parameters are. However, we still find it instructive to consider the effect of the various contributions in the S-T plane individually. The scatter plot shows the accessible region within the 95% CL ellipse of [14]. We find, as expected that the masses of the fourth quark generation can not be degenerate if they fulfill the constraints from the oblique parameters. The necessary mass difference is of the order of 50 GeV as stated in [11].



2.  $S - T$  test with ‘with leptons’ + ‘no  $V_{CKM}$ ’

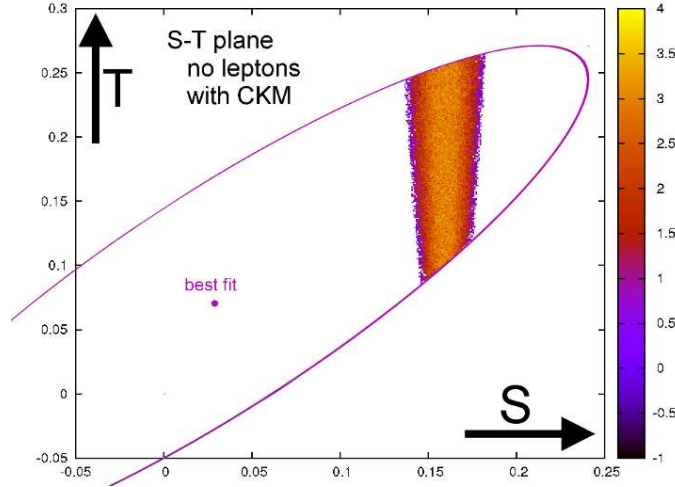
Next, we include the leptonic contributions (without lepton mixing) and still neglect CKM mixing. In this case also degenerate values of the quark masses of the fourth generation are in principle not excluded; however, this would require a significant mass gap in the lepton doublet to increase  $T$  (and preferably reduce  $S$ ).



The dark green points indicate values not in conflict with a degeneracy of the quark masses of the fourth generation.

3.  $S - T$  test with ‘no leptons’ + ‘with  $V_{CKM}$ ’

To study the “unperturbed” effect of a non trivial CKM structure of the fourth generation, let us discard again the leptons for the moment. The modified CKM structure results in an increase of the  $T$  parameter without changing  $S$ , cf. Eqs. (2.24) and (2.25).



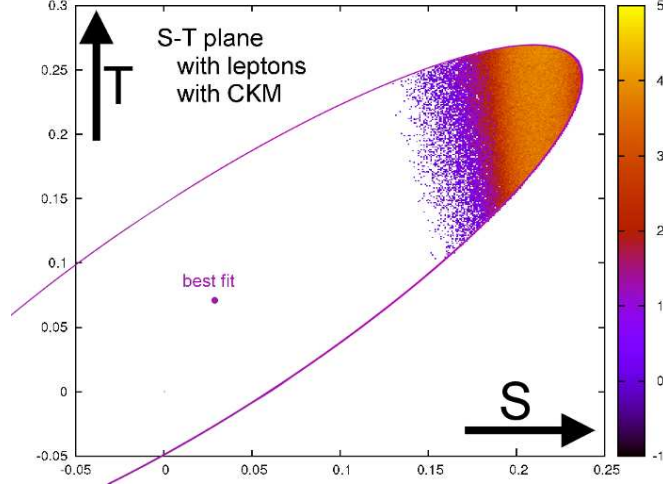
In this scenario an increase of  $T$  can originate either from a quark mass splitting or from nonzero mixing of the fourth generation with the SM quarks. However, a mass splitting must also induce a tiny (logarithmical) contribution to  $S$ ; so the central area which could not be reached in scenario ‘1’ corresponds to nonzero mixing and tiny mass splittings.

4.  $S - T$  test with ‘with leptons’ + ‘with  $V_{CKM}$ ’

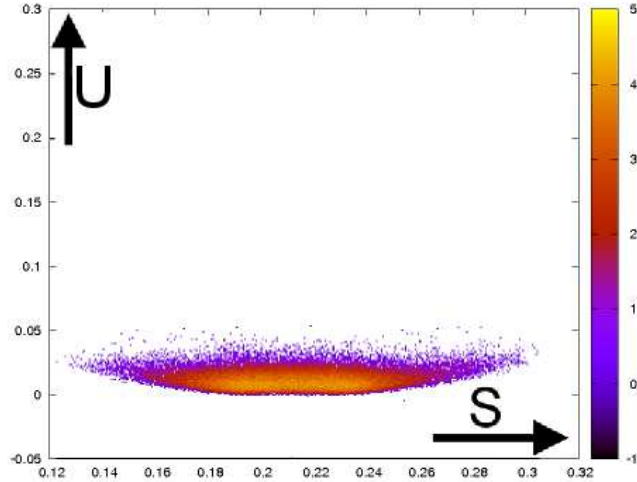
Here, we use the full expressions for  $S$  and  $T$  including both, leptons (without mixing) and CKM mixing. In this scenario we find a maximally allowed mass splitting of  $|m_{t'} - m_{b'}| < 80$  GeV for quarks and  $|m_{l_4} - m_{\nu_4}| < 140$  GeV for leptons.

Note that this splitting was also observed by the Gfitter group [97]; their fits also show

a minimal required mass splitting as they do not take the possible effects of a nontrivial CKM structure into account. Due to the effects of quark mixing, we do not find a lower bound for the splitting. In fact, a simultaneous degeneracy of quark and lepton masses is *not* excluded, even though the  $S$  parameter favors larger  $t'$  and  $l_4$  masses.



5. For completeness we also show the  $S$ - $U$  plane using the exact expression for  $U$  and  $S$ .



Summarizing our investigation of the  $S$ ,  $T$  and  $U$  parameters we get the following results:

- $U$  is not a priori small; only after the constraints on the quark mixing and the  $T$  parameter are used the maximal value for  $U$  is reduced below 0.06.
- The quarks of a 4th generation can be degenerated without violating the 95% CL constraints from electroweak precision observables<sup>7</sup>. However, this requires taking into account the effects of the non trivial flavor sector on  $T$ , i.e. mixing of the 4th generation fermions, or a sufficiently large mass splitting in the lepton sector. Note that, at first glance, this result seems to be in direct conflict with the standard statement that a degenerate fourth generation is excluded at the  $6\sigma$  level [74] by virtue of the  $S$

<sup>7</sup>For the 99% CL this statement will even hold stronger.

parameter. However, this statement always tacitly assumed a trivial CKM structure. The CKM factors in Eq. (2.25) can lead to  $T > 0$  even if both lepton and quark masses are degenerate. However, we did not investigate the effect of the  $Z \rightarrow b\bar{b}$  vertex, which tends to favor small or no mixing. Still one can conclude that the situation for tiny mass splittings or even degenerate masses drastically improves once mixing is taken into account.

Finally, we also have to (re)consider the contribution of the Higgs particle, since in the presence of a fourth family higher values of the Higgs mass may be possible [63]. The correction terms to the  $S$ ,  $T$  and  $U$  parameters read

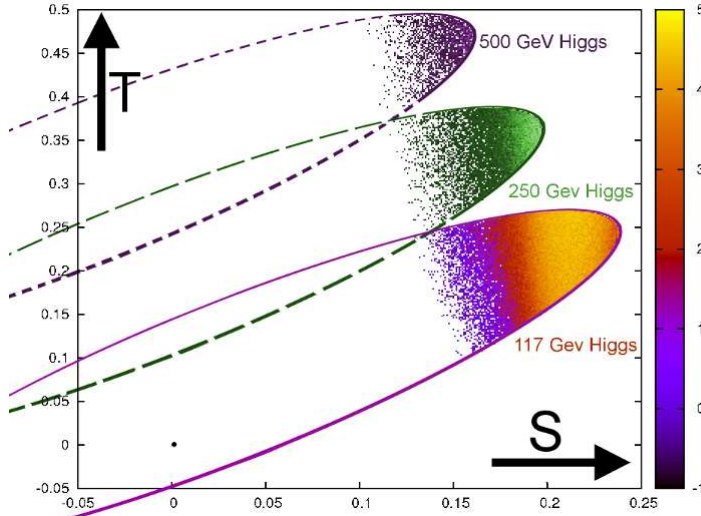
$$S_H = \frac{1}{12\pi} \ln \frac{M_H^2}{(117 \text{ GeV})^2}, \quad (2.33)$$

$$T_H = -\frac{3}{16\pi\bar{x}_W} \ln \frac{M_H^2}{(117 \text{ GeV})^2}, \quad (2.34)$$

$$U_H \approx 0. \quad (2.35)$$

Using that form for the Higgs contributions we implicitly subtract the used value for the Higgs mass in the fit (117 GeV) and add “our” value.

A heavier Higgs increases  $S$  and lowers  $T$ . Instead of adding  $S_H$  and  $T_H$  to our values for  $S$  and  $T$ , we subtracted the Higgs contributions from the fit values to make the diagram easier to understand. So we shift the ellipse and not our data sets. We investigate the  $S$  and  $T$  parameters for three values of the Higgs mass: 117 GeV, 250 GeV and 500 GeV.



The decrease in  $T$  is welcome, as it allows even bigger mass splitting (or alternatively larger mixing); however, the simultaneous increase of  $S$  due to the heavy Higgs completely seems to neutralize or even reverse this effect. Hence, as stated recently in [14] very large values for the mass of a SM-like Higgs are clearly not favored. However, for a 250 GeV Higgs scenario the origin (SM3) is outside the ellipse, whereas some SM4 points are inside and thus more likely.



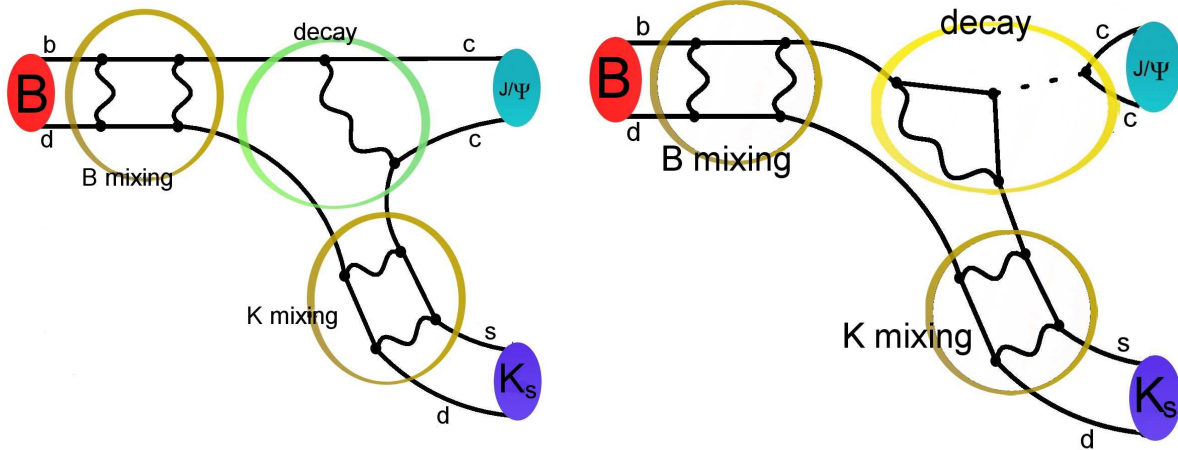


Figure 2: Schematic diagrams for the necessary ingredients of the golden plated channel  $B_d \rightarrow J/\Psi K_s$ : B-mixing, Kaon mixing and the decay process itself. The left panel shows the tree-level and the right panel the penguin mediated decay. The dashed line represents any current capable of creating a  $J/\Psi$ , e.g. two gluons.

### 2.3 Flavor physics constraints — FCNC processes

After addressing the electroweak bounds, we turn to the constraints imposed by precision observables of flavor physics involving a FCNC. One can hope to impose severe constraints on the model by utilizing information from such processes as it is well known that the weak interaction bypasses the Appelquist-Carazzone decoupling theorem [98]; thus, FCNC processes are very sensitive to contributions of new physics.

However, the selection of flavor physics bounds on a hypothetical fourth family is a non trivial issue. The reason for this is the fact that some processes known for being theoretically or experimentally very clean, may in fact specifically require the SM3 setup. Hence, it is always necessary to check, if a specific feature (of the SM3) crucial e.g. for the data analysis is preserved in the fourth generation extension. If this is not the case, it may either be necessary to repeat the analysis without some SM3 simplification — much like the need to give up  $3 \times 3$  unitarity — or the whole process may not even be feasible anymore.

As an example for how unexpected complications may arise (see also [99] for a more detailed discussion), we discuss the so-called golden plated channel for the determination of the Standard Model CKM angle  $\beta$ :  $B_d \rightarrow J/\Psi K_s$  [100]. This channel is renowned for being theoretically very clean (in the SM3). Since the decay process is tree level dominated, it is usually taken for granted that the contribution of the fourth generation quarks to the decay is generally small. Therefore the SM4 could, in principle, be an explanation for discrepancies of the measurement of  $\sin(2\beta)$  in the  $B_d \rightarrow J/\Psi K_s$  and  $B_d \rightarrow \Phi K_s$ , as  $B_d \rightarrow \Phi K_s$  is penguin dominated and as such more sensitive to new physics effects, see [101] for a more detailed version of this argument. However, it turns out that this elegant picture of the consequences of the fourth generation is, unfortunately, too simple. The reason for this is the following:  $\sin(2\beta)$  is extracted via time-dependent CP asymmetries. The necessary ingredients are (i)  $B_d$  mixing, (ii) Kaon mixing and the (iii) decay process itself, see Fig. 2 for a schematic picture of the relevant subprocesses. There are in fact two decay processes, the tree-level decay and the top mediated penguin decay ( $c$  and  $u$  penguin are expected to be tiny). However, the beauty



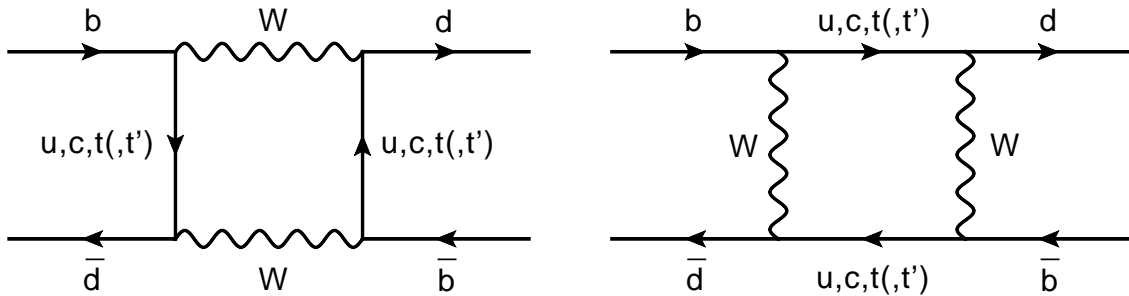
of this process in the SM3 is that the tree level decay and the  $t$  penguin have (to a fantastic accuracy) the same CKM phase. Hence, it is not necessary to take into account e.g. different hadronization effects as they will only modify the overall amplitude but not the phase. Adding an additional generation has two new, separate effects. First of all, the expressions for the box diagram changes (see formulae below), so that additional CKM factors contribute; therefore, instead of the CKM angle  $\beta$  a different combination of CKM angles can be extracted from this process. This is, however, not a problem as one could still use  $B_d \rightarrow J/\Psi K_s$  to constrain the SM4. The real problem is the simultaneous modification of the penguin diagram by the  $t'$  loop. As the  $t'$  will introduce some new virtually unconstrained phase, penguin and tree decay now have *different* CKM phases; the fourth generation introduces a mismatch between tree and penguin decay, which makes taking into account hadronization and QCD corrections mandatory. Therefore, it is not clear what quantity can be extracted from time-dependent CP asymmetries in  $B_d \rightarrow J/\Psi K_s$  in the SM4 scenario. This, of course, limits the usefulness of this process for constraining the parameters of the model.

As the above example shows, not all processes can be used to obtain limits on the parameters of the SM4 and one has to be careful not to make use of a bound whose experimental input essentially requires the SM3 setup or some SM3 specific feature.

### 2.3.1 FCNC constraints with no sensitivity to lepton mixing

Another issue is the sensitivity of some processes to the properties of the leptons. Since we do not take mixing in the lepton sector into account and in fact assume lepton number conservation, quantities that are insensitive to the precise structure of the lepton sector are of course advantageous.

**We first consider the mixing of the  $K$ -, the  $D$ -, the  $B_d$ - and the  $B_s$ -system.** For completeness we repeat the relevant formulae already given in [50]. The virtual part of the box diagram (here e.g. for  $B_d$ -mixing)



is encoded in  $M_{12}$ , which is very sensitive to new physics contributions. It is related to the mass difference of the heavy and light neutral mass eigenstate via

$$\Delta M = M_{B_H} - M_{B_L} = 2|M_{12}|. \quad (2.36)$$

In the SM3 one obtains the following relations

$$M_{12}^{K^0} \propto \eta_{cc} \left( \lambda_c^{K^0} \right)^2 S_0(x_c) + 2\eta_{ct} \lambda_c^{K^0} \lambda_t^{K^0} S(x_c, x_t) + \eta_{tt} \left( \lambda_t^{K^0} \right)^2 S_0(x_t), \quad (2.37)$$

$$M_{12}^{B_d} \propto \eta_{tt} \left( \lambda_t^{B_d} \right)^2 S_0(x_t), \quad (2.38)$$

$$M_{12}^{B_s} \propto \eta_{tt} \left( \lambda_t^{B_s} \right)^2 S_0(x_t), \quad (2.39)$$

with the Inami-Lim functions [102]

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln[x]}{2(1-x)^3}, \quad (2.40)$$

$$S(x, y) = xy \left[ \frac{1}{y-x} \left( \frac{1}{4} + \frac{3}{2} \frac{1}{1-y} - \frac{3}{4} \frac{1}{(1-y)^2} \right) \ln[y] \right. \\ \left. + \frac{1}{x-y} \left( \frac{1}{4} + \frac{3}{2} \frac{1}{1-x} - \frac{3}{4} \frac{1}{(1-x)^2} \right) \ln[x] - \frac{3}{4} \frac{1}{1-x} \frac{1}{1-y} \right], \quad (2.41)$$

where  $x_{c,t} = \frac{m_{c,t}^2}{M_W^2}$ , the CKM elements

$$\lambda_x^{K^0} = V_{xd} V_{xs}^*, \quad \lambda_x^{B_d} = V_{xd} V_{xb}^*, \quad \lambda_x^{B_s} = V_{xs} V_{xb}^* \quad (2.42)$$

and the QCD corrections [103, 104, 105]

$$\eta_{cc} = 1.38 \pm 0.3, \quad \eta_{ct} = 0.47 \pm 0.04, \quad \eta_{tt} = 0.5765 \pm 0.0065. \quad (2.43)$$

The full expressions for  $M_{12}$  can be found e.g. in [103, 106]. In deriving these expressions the unitarity of the  $3 \times 3$  matrix was explicitly used, i.e.

$$\lambda_u^X + \lambda_c^X + \lambda_t^X = 0. \quad (2.44)$$

Moreover, in the  $B$ -system the CKM elements of the different internal quark contributions are all roughly of the same size. Only the top contribution, which has by far the largest value of the Inami-Lim functions, survives. This is not the case in the  $K$ -system. Here the top contribution is CKM suppressed, while the kinematically suppressed charm terms are CKM favored. Therefore, both have to be taken into account. More information about the mixing of neutral mesons can be found e.g. in [106, 107].

We define the parameter  $\Delta$  as the ratio of the new physics model prediction (in our case SM4) for a generic observable to the SM3 theory value; thus, it quantifies the deviation from the standard model [106]. For  $M_{12}$  one would then define:

$$\Delta := \frac{M_{12}^{SM4}}{M_{12}^{SM3}} = |\Delta| e^{i\phi\Delta}. \quad (2.45)$$

This representation is convenient as one can effectively map any observable to the complex  $\Delta$  plane; this allows a straightforward comparison of the sensitivity of the various observables to the effect of the model. Experimental data can be mapped analogously by plotting  $\hat{\Delta} :=$

$O^{Exp}/O^{SM3}$  in the same complex plane.  $O^{Exp}$  denotes the experimental value of a generic observable and  $O^{SM3}$  the theory prediction within the SM3. In the SM4, we obtain

$$M_{12}^{K^0, SM4} \propto \eta_{cc} \left( \lambda_c^{K^0} \right)^2 S_0(x_c) + 2\eta_{ct} \lambda_c^{K^0} \lambda_t^{K^0} S(x_c, x_t) + \eta_{tt} \left( \lambda_t^{K^0} \right)^2 S_0(x_t) \quad (2.46)$$

$$+ 2\eta_{ct'} \lambda_c^{K^0} \lambda_{t'}^{K^0} S(x_c, x_{t'}) + 2\eta_{tt'} \lambda_t^{K^0} \lambda_{t'}^{K^0} S(x_t, x_{t'}) + \eta_{t't'} \left( \lambda_{t'}^{K^0} \right)^2 S_0(x_{t'}),$$

$$M_{12}^{B_d, SM4} \propto \eta_{tt} \left( \lambda_t^{B_d} \right)^2 S_0(x_t) + \eta_{t't'} \left( \lambda_{t'}^{B_d} \right)^2 S_0(x_{t'}) + 2\eta_{tt'} \lambda_t^{B_d} \lambda_{t'}^{B_d} S(x_t, x_{t'}), \quad (2.47)$$

$$M_{12}^{B_s, SM4} \propto \eta_{tt} \left( \lambda_t^{B_s} \right)^2 S_0(x_t) + \eta_{t't'} \left( \lambda_{t'}^{B_s} \right)^2 S_0(x_{t'}) + 2\eta_{tt'} \lambda_t^{B_s} \lambda_{t'}^{B_s} S(x_t, x_{t'}). \quad (2.48)$$

Note that now also those CKM elements change that describe the mixing within the first three families! For simplicity we take the new QCD corrections to be (see also [66, 67])

$$\eta_{t't'} = \eta_{tt'} = \eta_{tt} \text{ and } \eta_{ct'} = \eta_{ct}. \quad (2.49)$$

It is interesting to note here that the only information we have currently about the CKM elements  $V_{td}$  and  $V_{ts}$  comes from  $B$ - and  $K$ -mixing plus assuming the unitarity of  $V_{CKM3}$ .

**The mass difference in the neutral  $D$  system** can be used to infer a very strong bound on  $|V_{ub}V_{cb}|$ , see [108]<sup>8</sup>. We redid this analysis in [50] and softened the bound. The mass difference in the neutral  $D^0$ -system is typically expressed in terms of the parameter  $x_D$ :

$$x_D = \frac{\Delta M_D}{\Gamma_D} \leq \frac{2|M_{12}^{D^0}|}{\Gamma_D}. \quad (2.50)$$

For more information on the last inequality see e.g. the discussion in [110, 111]. HFAG [112] quotes for the experimental value of  $x_D$

$$x_D = (0.811 \pm 0.334) \cdot 10^{-2}. \quad (2.51)$$

The main difference compared to the above discussed  $K$ - and  $B$ -mixing systems is that in the  $D$ -system the theory prediction in the SM3 is theoretically not well under control, see e.g. [110, 111]. However, the pure contribution of a heavy fourth generation to  $M_{12}$  can be calculated reliably. Using the unitarity of the CKM matrix of the SM4  $\lambda_d^{D^0} + \lambda_s^{D^0} + \lambda_b^{D^0} + \lambda_{b'}^{D^0} = 0$  (with  $\lambda_x^{D^0} = V_{cx}V_{ux}^*$ ), the full expression for  $M_{12}$  reads

$$\begin{aligned} M_{12}^{D^0} \propto & \left( \lambda_s^{D^0} \right)^2 S_0(x_s) + 2\lambda_s^{D^0} \lambda_b^{D^0} S(x_s, x_b) + \left( \lambda_b^{D^0} \right)^2 S_0(x_b) + \text{LD} \\ & + 2\lambda_s^{D^0} \lambda_{b'}^{D^0} S(x_s, x_{b'}) + 2\lambda_b^{D^0} \lambda_{b'}^{D^0} S(x_b, x_{b'}) + \text{LD} \\ & + \left( \lambda_{b'}^{D^0} \right)^2 S_0(x_{b'}), \end{aligned} \quad (2.52)$$

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<sup>8</sup>A similar strategy was recently used in [109].

where the proportionality constant is

$$\frac{G_F^2 M_W^2 M_D}{12\pi^2} f_D^2 B_D \eta(m_c, M_W). \quad (2.53)$$

We use the same numerical values as in [50]. The first line of (2.52) corresponds to the pure SM3 contribution, the third line is due to contributions of the heavy 4th generation and the second line is a term arising when SM3- and  $b'$  contributions mix:

$$M_{12}^{D^0} = M_{12,SM3}^{D^0} + M_{12,Mix}^{D^0} + M_{12,b'}^{D^0}. \quad (2.54)$$

The idea of [108] was to neglect all terms in  $M_{12}^{D^0}$ , except  $M_{12,b'}^{D^0}$ , and to equate this term with the experimental number for  $x_D$ , since all perturbative short-distance contributions with light internal quarks are negligible. Since it is not completely excluded that there might be large non-perturbative contributions to both  $M_{12,SM3}^{D^0}$  and  $M_{12,Mix}^{D^0}$  (denoted by  $LD$ ), each of the size of the experimental value of  $x_D$ , we get the following bound

$$3M_{12}^{D^0,Exp} \geq M_{12,b'}^{D^0}. \quad (2.55)$$

Allowing this possibility we obtain the following bounds on  $|V_{ub'}V_{cb'}|$

$$|V_{ub'}V_{cb'}| \leq \begin{cases} 0.00395 & \text{for } m_{b'} = 200 \text{ GeV}, \\ 0.00290 & \text{for } m_{b'} = 300 \text{ GeV}, \\ 0.00193 & \text{for } m_{b'} = 500 \text{ GeV}. \end{cases} \quad (2.56)$$

This bound is still by far the strongest direct constraint on  $|V_{ub'}V_{cb'}|$ .

**Next, we consider the  $b \rightarrow s\gamma$  transition.** In [50] we approximated the treatment of the FCNC decay  $b \rightarrow s\gamma$  by simply looking at the product of CKM structure and the corresponding Inami-Lim function  $D'_0(x_t)$  [102]<sup>9</sup>.

$$\Delta_{b \rightarrow s\gamma} := \frac{|\lambda_t^{SM4}|^2 D'_0(x_t)^2 + 2\text{Re}(\lambda_t^{SM4} \lambda_{t'}^{SM4}) D'_0(x_t) D'_0(x_{t'}) + |\lambda_{t'}^{SM4}|^2 D'_0(x_{t'})^2}{|\lambda_t^{SM3}|^2 D'_0(x_t)^2}, \quad (2.57)$$

with

$$D'_0(x) = -\frac{-7x + 5x^2 + 8x^3}{12(1-x)^3} + \frac{x^2(2-3x)}{2(1-x)^4} \ln[x] \quad (2.58)$$

and  $\lambda_x \equiv \lambda_x^{B_s}$ . We assumed that parameters which give a value of  $\Delta_{b \rightarrow s\gamma}$  close to one will also lead only to small deviations of  $\Gamma(b \rightarrow s\gamma)^{SM4}/\Gamma(b \rightarrow s\gamma)^{SM3}$  from one. However, this crude treatment imposed a too strong bound on the 3 – 4 mixing.

In this work we will use the full leading logarithmic expression for  $b \rightarrow s\gamma$ , see also [66, 67]. Following [113] we normalize the  $b \rightarrow s\gamma$  decay rate to the semi leptonic decay rate

$$R := \frac{\Gamma(b \rightarrow s\gamma)}{\Gamma(b \rightarrow ce\bar{\nu})} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{6\alpha}{\pi f(z)} \left| C_{7\gamma}^{(0)eff} \right|^2, \quad (2.59)$$

$f(z = m_c^2/m_b^2)$  is a phase space factor, which we will not need later on. It is interesting to note that in deriving this formula the unitarity of the  $3 \times 3$  CKM matrix was already used

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<sup>9</sup>The Inami-Lim function  $D'_0(x_t)$  is proportional to the Wilson-coefficient  $C_{7\gamma}(M_W)$ .

and the CKM combination  $\lambda_u = V_{us}^* V_{ub}$  was neglected (in comparison to  $\lambda_c$  and  $\lambda_t$ ). The effective Wilson coefficient  $C_{7\gamma}^{(0)eff}$  is a linear combination of the penguin Wilson coefficients  $C_7$  and  $C_8$ , which are accompanied by the CKM structure  $\lambda_t$  and the current-current Wilson coefficient  $C_2$  with the corresponding CKM structures  $\lambda_c$  and  $\lambda_u$

$$C_{7\gamma}^{(0)eff}(\mu) = C_7^{eff1}(\mu) + C_7^{eff2}(\mu) + C_7^{eff3}(\mu), \quad (2.60)$$

$$C_7^{eff1}(\mu) = \eta^{\frac{16}{23}} C_{7\gamma}^{(0)}(M_W), \quad (2.61)$$

$$C_7^{eff2}(\mu) = \frac{8}{3} \left( \eta^{\frac{14}{23}} - \eta^{\frac{16}{23}} \right) C_{8g}^{(0)}(M_W), \quad (2.62)$$

$$C_7^{eff3}(\mu) = \sum_{i=1}^8 h_i \eta^{a_i} C_2^{(0)}(M_W), \quad (2.63)$$

with

$$\eta(\mu) := \frac{\alpha_s(M_W)}{\alpha_s(\mu)}. \quad (2.64)$$

The values for  $h_i$  and  $a_i$  are given in Table XXVII of [113]. The initial conditions of the Wilson coefficients read

$$C_2^{(0)}(M_W) = 1, \quad (2.65)$$

$$C_{7\gamma}^{(0)}(M_W) = -\frac{1}{2} D'_0 \left( x_t = \frac{m_t^2}{M_W^2} \right), \quad (2.66)$$

$$C_{8g}^{(0)}(M_W) = -\frac{1}{2} E'_0 \left( x_t = \frac{m_t^2}{M_W^2} \right). \quad (2.67)$$

$D'_0(x)$  is given above in Eq. (2.58),  $E'_0(x)$  reads

$$-\frac{1}{2} E'_0(x) = -\frac{2x + 5x^2 - x^3}{8(1-x)^3} - \frac{3x^2}{4(1-x)^4} \ln[x]. \quad (2.68)$$

Numerically it turns out that even for large values of  $m_t$  (up to 1000 GeV)  $C_7^{eff3}$  is the dominant contribution to  $C_{7\gamma}^{(0)eff}(\mu)$ . In [50] we have only taken  $C_7^{eff1}$  into account and therefore overestimated the effects of a fourth generation to the branching ratio of the decay  $b \rightarrow s\gamma$ . Putting everything together we get

$$\Delta_{b \rightarrow s\gamma} := \frac{R^{\text{SM3}}}{R^{\text{SM4}}} \quad (2.69)$$

$$= \left| \frac{V_{cb}^{\text{SM3}}}{V_{cb}^{\text{SM4}}} \right|^2 \left| \frac{\lambda_c^{\text{SM4}} C_7^{eff3} - \lambda_t^{\text{SM4}} (C_7^{eff1} + C_7^{eff2}) - \lambda_{t'}^{\text{SM4}} (C_7^{eff1'} + C_7^{eff2'})}{\lambda_c^{\text{SM3}} C_7^{eff3} - \lambda_t^{\text{SM3}} (C_7^{eff1} + C_7^{eff2})} \right|^2. \quad (2.70)$$

In [67] it was suggested to use the LO expression for  $b \rightarrow s\gamma$  at a low scale of  $\mu = 3.22$  GeV in order to reproduce the numerical value of the NLO expression. We have checked that  $\Delta_{b \rightarrow s\gamma}$  is quite insensitive to a variation of the scale between  $m_b$  and 3 GeV, so we use  $\mu = m_b$ .

### 2.3.2 FCNC constraints with sensitivity to lepton mixing

Next we also discuss FCNC processes that are sensitive to lepton mixing. In principle lepton mixing has to be investigated in the same manner as the quark mixing. For simplicity we have neglected lepton mixing in this paper. However, we will take into account the conservative bounds on  $V_{e\nu_4}$ ,  $V_{\mu\nu_4}$  and  $V_{\tau\nu_4}$  given in [45] for the rare decay  $B_s \rightarrow \mu^+\mu^-$ . In the ratio of the SM4 and SM3 predictions for the branching ratios almost everything cancels out and one is left with the product of CKM elements and Inami-Lim functions

$$\begin{aligned}\Delta_{B_s \rightarrow \mu\mu} &:= \frac{BR(B_s \rightarrow \mu^+\mu^-)^{\text{SM4}}}{BR(B_s \rightarrow \mu^+\mu^-)^{\text{SM3}}} \\ &= \frac{|\lambda_t^{\text{SM4}} Y_0(x_t) + \lambda_{t'}^{\text{SM4}} Y_0(x_{t'})|^2}{|\lambda_t^{\text{SM3}} Y_0(x_t)|^2},\end{aligned}\tag{2.71}$$

with the Inami-Lim function

$$Y_0[x] = \frac{x}{8} \left( \frac{x-4}{x-1} + 3 \frac{x}{(x-1)^2} \ln[x] \right).\tag{2.72}$$

Including also the leptonic contributions we have to make the following substitutions [67] in Eq. (2.71)

$$Y_0(x_t) \rightarrow Y_0(x_t) - |U_{\mu 4}|^2 S(x_t, x_{\nu_4}),\tag{2.73}$$

$$Y_0(x_{t'}) \rightarrow Y_0(x_{t'}) - |U_{\mu 4}|^2 S(x_{t'}, x_{\nu_4}),\tag{2.74}$$

where  $S$  is the box function given in Eq. (2.41),  $x_{\nu_4}$  is given by the mass of the fourth neutrino and  $U_{\mu 4}$  is the PMNS matrix element describing the mixing between the  $\mu$  and the fourth neutrino. In [45] the bound  $U_{\mu 4} < 0.029$  was derived. Using this information we find that leptonic contributions give at most a relative correction of 0.5 per mille, so we can safely neglect them.

The branching ratio for  $B_s \rightarrow \mu\mu$  is not measured yet, HFAG quotes [114] (for the current experimental bound from TeVatron, see also [115])

$$Br(B_s \rightarrow \mu^+\mu^-) < 3.6 \cdot 10^{-8}.\tag{2.75}$$

In the SM3 one expects a value of [67]

$$Br(B_s \rightarrow \mu^+\mu^-) = (3.2 \pm 0.2) \cdot 10^{-9}.\tag{2.76}$$

### 2.3.3 Allowed ranges for the $\Delta$ parameters

Now we come to a crucial point: the fixing of the allowed ranges for the values of the different  $\Delta$ s. For our exploratory study - in comparison to a full fit that will be performed in future - we fix reasonable ranges for the  $\Delta$ s. Therefore we have to investigate theoretical and experimental errors. The FCNC quantities  $\Delta M_s$ ,  $\Delta M_d$  and  $b \rightarrow s\gamma$  are dominated by theoretical uncertainties. Currently, in particular the hadronic uncertainties are under

intense discussion, see e.g. [116]. Therefore, we use conservative estimates for the theoretical errors.

	Bound
$ \Delta_{B_d} $	$1 \pm 0.3$
$\phi_{B_d}^\Delta$	$0 \pm 10^\circ$
$ \Delta_{B_s} $	$1 \pm 0.3$
$\phi_{B_s}^\Delta$	free
$\text{Re}(\Delta_K)$	$1 \pm 0.5$
$\text{Im}(\Delta_K)$	$0 \pm 0.3$
$\Delta_{b \rightarrow s \gamma}$	$1 \pm 0.15$
$\Delta_{B_s \rightarrow \mu \mu}$	$< 15$

Since we choose for the central values of our  $\Delta$ s the value one, all resulting allowed parameter points for  $V_{CKM4}$  will be scattered around the SM3 values by definition. For a future fit we will use  $\Delta$ s with the central value  $\tilde{\Delta} = O^{Exp}/O^{SM3}$ .

This means in other words that we do not take into account some current deviations in flavor physics in our current analysis, we simply include them in our error band for the  $\Delta$ s.

### 3 Putting things together—Constraints on the parameter space

In order to constrain the mixing with the fourth quark family we perform a scan through the parameter space of the model. To this end we use the exact parameterization of  $V_{CKM4}$  described in Sec. 2.1, Eq. (2.3). For the tree-level bounds we use the central values and standard deviations as given in 2.2.1; we allow for a variation at the  $2\sigma$  level. The restrictive Peskin-Takeuchi parameters are allowed to vary at the 99% confidence level of [14], cp. Sec. 2.2.3. For the quark masses we use a hard lower limit of 280 GeV and allow for a maximal mass difference of 80 GeV as determined in Sec. 2.2.3. The lepton masses are chosen to be larger than 100 GeV with a maximal splitting of 140 GeV. For the FCNC we use the  $\Delta$ s given in Sec 2.3.3.

Then we generate a large number ( $\mathcal{O}(10^{11})$ ) of randomly distributed points in the 13 dimensional parameter space<sup>10</sup>. For each point we determine the value of CKM matrix elements, flavor and electroweak observables and check whether the various experimental bounds are passed (for details see [50]).

#### 3.1 Result for the mixing angles

Let us for the moment ignore correlations among the various parameters and focus on the maximally allowed size of the mixing with the fourth generation. Table 1 shows the limits on the mixing angles  $\theta_{14}$ ,  $\theta_{24}$ , and  $\theta_{34}$  — without and with the electroweak bounds.

As already expected in Sec. 2, the Peskin-Takeuchi parameters impose strong constraints on the mixing of Standard Model fermions with the fourth generation. These numbers are comparable with the ones quoted in [62, 67]. The most dramatic effect is observed for the mixing angle  $\theta_{34}$ . The maximal size is roughly halved by the virtue of the  $T$  parameter alone. So, already at this stage, one is able to conclude that a study of the flavor aspects of SM4

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<sup>10</sup> $m_{t'}, m_{b'}, m_{l_4}, m_{\nu_4}, \theta_{12}, \theta_{23}, \theta_{13}, \theta_{14}, \theta_{24}, \theta_{34}, \delta_{13}, \delta_{14}, \delta_{24}$ .



	only tree-level	with FCNC bounds	with electroweak observables
$\theta_{14}$	$< 0.07$	$< 0.0535$	$< 0.0535$
$\theta_{24}$	$< 0.19$	$< 0.145$	$< 0.121$
$\theta_{34}$	$< 0.8$	$< 0.67$	$< 0.35$

Table 1: Maximal mixing of SM fermion generations with the fourth generation. The left column show the effect of tree-level bounds alone, for the central column FCNC bounds where added and the right-most column gives the limits including electroweak parameters.

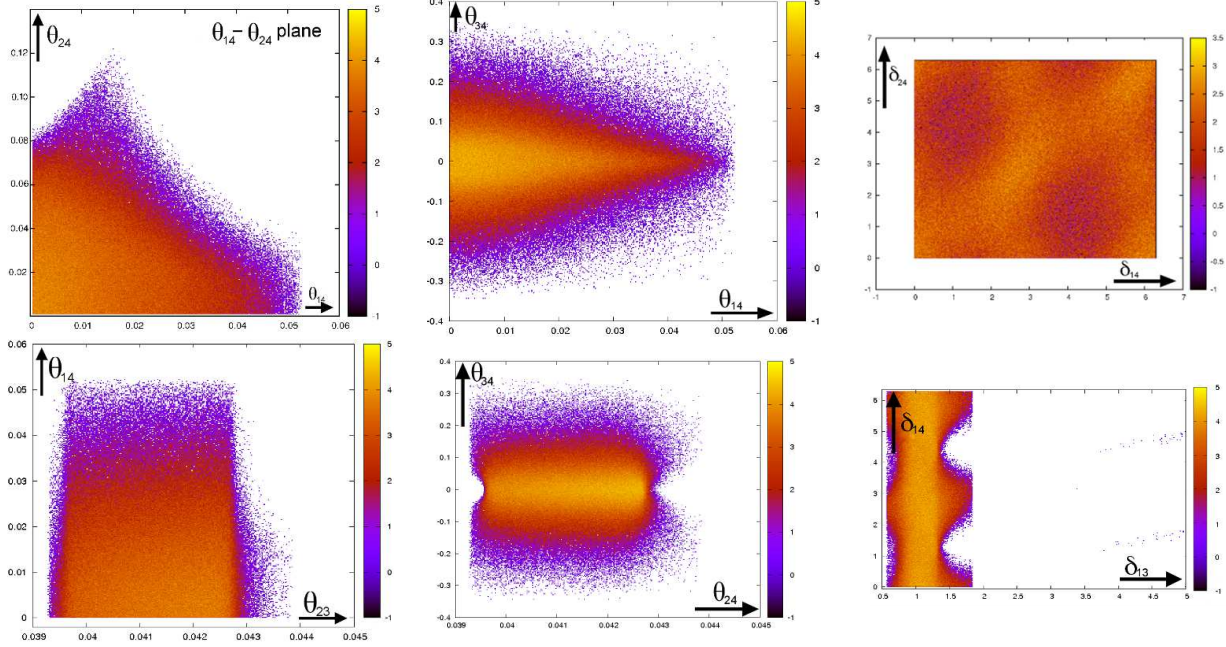
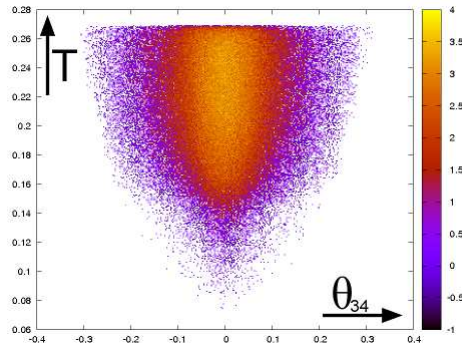


Figure 3: Some correlations of the angles and phases.

must not be decoupled from a simultaneous analysis of the electroweak sector.

To illustrate the dependence of  $T$  on  $\theta_{34}$  we also show the scatter plot for  $T$  versus  $\theta_{34}$ :



Next, the correlations between the different angles are examined. The results are depicted in Fig. 3. Obviously, the maximal mixing angles given in Table 1 cannot be simultaneously realized; especially  $\theta_{14}$  and  $\theta_{24}$  show a rather strong correlation and maximal  $\theta_{24}$  is only possible for  $\theta_{14}$  close to 0.018. Note e.g. the  $\theta_{14} - \theta_{34}$  correlation; simultaneous large mixing angles  $\theta_{34}$  and  $\theta_{14}$  are also disfavored. Indeed, this observation is rather natural, as  $V_{td}$

includes a term  $c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}}$ . Hence, simultaneous large  $s_{14}$  and  $s_{34}$  would lead to a large modification of  $V_{td}$ ;  $B_d$  mixing would be sensitive to this and indeed proves to be the most restrictive of the mixing observables. Due to this observation we disfavor a strategy based on starting from fixed bounds on the mixing angles without taking the correlations into account.

The large ‘voids’ in the  $\delta_{13} - \delta_{14}$  plane can be traced to the effect of the direct limit on the phases due to the CKM angle  $\gamma$ .

### 3.2 Results for the CKM elements

Since the  $3 \times 3$  unitarity fixes the values of the second and third row CKM elements rather precisely in the SM3, it is interesting to see the effect of the lifting of the unitarity constraint on  $V_{cd}$ ,  $V_{cs}$ ,  $V_{cb}$ ,  $V_{td}$ ,  $V_{ts}$  and  $V_{tb}$ . In Figure 4 we present the possible values for these CKM matrix elements in the complex plane. Note that a CKM matrix element itself is not a physical observable as it depends on phase conventions and CKM parameterization; one can, however, compare the values for the elements once the representation and phase convention is fixed. The plots correspond to the standard representation, cp. (2.1), which is the limit of the Botella-Chau representation for zero mixing with a fourth family.

The absolute value of the elements of the second row cannot change much with respect to the SM3; however, it is interesting to observe that the imaginary part of  $V_{cd}$  and  $V_{cs}$  can be increased by an order of magnitude. This might be potentially interesting for searches for CP violation in the charm sector. While  $V_{cb}$  does not have an imaginary part in the standard representation in SM3, a tiny imaginary part can be present in SM4.

The absolute value of both,  $V_{td}$  and  $V_{ts}$ , can be modified (with respect to their SM3 value) by approximately a factor 2. More important, the imaginary part of  $V_{ts}$  can be an order of magnitude larger than in the SM. Therefore, one can expect that the weak phases of processes involving  $V_{ts}$ , e.g.  $B_s$  mixing, may experience large corrections.

The absolute value of  $V_{tb}$  can be as low as 0.93. Without the constraints coming from oblique parameters this limit would be much lower — around 0.8. This again shows that the electroweak sector imposes strong limits on the flavor structure of SM4.

This number is in particular interesting since it can be compared with direct determinations of  $V_{tb}$  from Single-top-production from TeVatron [117, 118, 119]

$$V_{tb}^{\text{TeVatron}} = 0.88 \pm 0.07. \quad (3.77)$$

### 3.3 New physics in $B_s$ -mixing

The results for the complex  $\Delta_{B_s}$  plane is particularly interesting since there might be some hints on new physics effects in the CP-violating phase of  $B_s$  mixing, see [106, 120] and the web-updates of [82]. In [106] a visualization of the combination of the mixing quantities  $\Delta M_s$ ,  $\Delta\Gamma_s$ ,  $a_{sl}^s$ , which are known to NLO-QCD [103, 121, 122, 123] and of direct determinations of  $\Phi_s$  in the complex  $\Delta$ -plane was suggested. Combining recent measurements [114, 124] for the phase  $\Phi_s$  one obtains a deviation from the tiny SM-prediction [106] in the range of 2 to 3  $\sigma$ :

- HFAG: 2.2  $\sigma$  [114],
- CKMfitter: 2.1...2.5  $\sigma$  [125, 126],

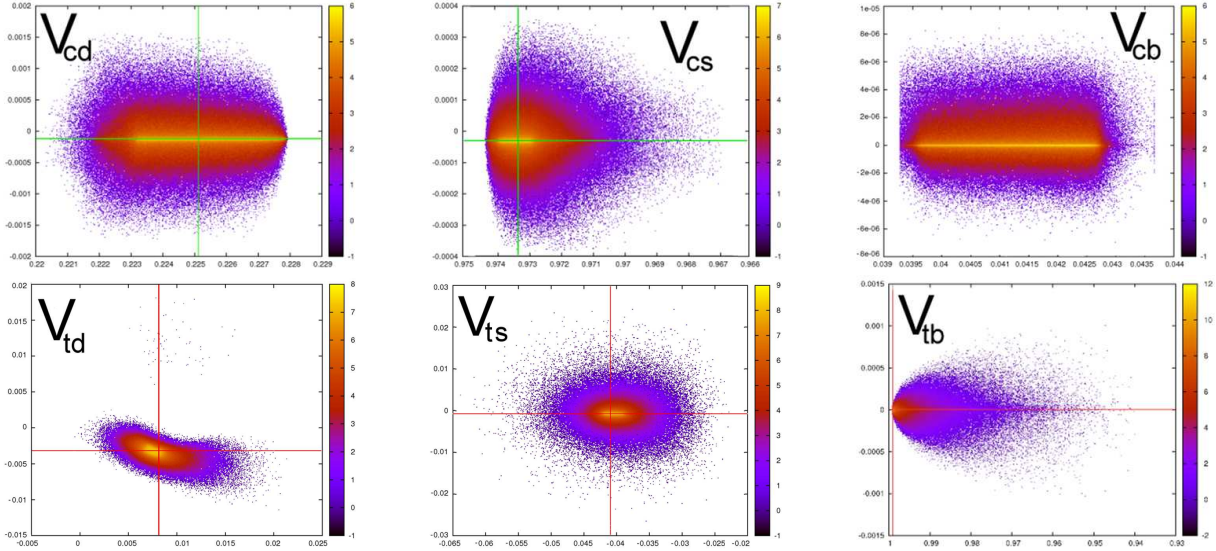


Figure 4: Possible modifications of the SM3 CKM matrix elements  $V_{cd}$ ,  $V_{cs}$ ,  $V_{cb}$ ,  $V_{td}$ ,  $V_{ts}$  and  $V_{tb}$  in the SM4 scenario. Depicted is the real part versus the imaginary part of the CKM element (in the standard representation). The crossed lines show the SM3 value;  $V_{cb}$  is real by construction in the SM3.

- UTfit:  $2.9 \sigma$  [120].

The central values of these deviations cluster around

$$\Phi_s \approx -51^\circ. \quad (3.78)$$

Very recently the D0 collaboration announced a  $3.2\sigma$  deviation of a linear combination of  $a_{sl}^d$  and  $a_{sl}^s$  [127] from the standard model prediction in [106]. This deviation also indicates a large negative value of  $\Phi_s$ .

As can be read off from the left picture of Figure 5 sizeable values for  $\Phi_s$  can also be obtained in scenarios with additional fermions. Such large values for  $\Phi_s$  are not favored, but they are possible. An enhancement of  $\Phi_s$  to large negative values by contributions of a fourth generation was first predicted in [40], by choosing the parameters of the fourth generations in such a way that other flavor problems like the  $B \rightarrow K\pi$  puzzle are solved. Once the T parameter is implemented with full CKM dependence and used as an additional bound on the CKM elements, a large value of  $\Phi_s$  seems to be very unlikely and requires a significant fine tuning of the parameters, see the right picture of Figure 5. In that respect we differ slightly from the conclusion of e.g. [40, 66, 67, 128], where very large values for  $\Phi_s$  are allowed<sup>11</sup>.

Here we expect new and considerably more precise data from TeVatron and LHC soon. If the central value stayed at the current position, the possibility would arise to find new physics that can not originate from an additional fermion family alone.

It is interesting to study the mass dependence of the phase  $\Phi_s$ . Already in [128] it was noted that large phases clearly favor small  $t'$  (and  $b'$ ) masses and the largest phases require a value of  $m_{t'}$  close to 300 GeV; this behaviour is also present in our analysis and it seems

<sup>11</sup>If we also described the problems in e.g.  $B \rightarrow K\pi$  by a fourth family, we also would exclude the points with  $\Phi_s$  close to zero and we would predict a sizeable phase - around  $-20^\circ$  - but no points with e.g.  $-50^\circ$  survive in our analysis. Because of hadronic uncertainties we did not include  $B \rightarrow K\pi$  in our analysis.

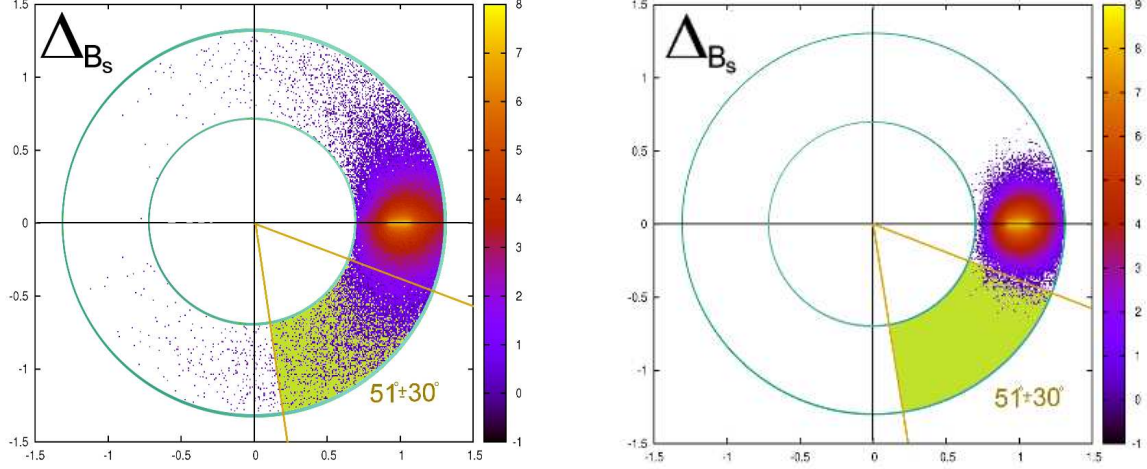


Figure 5: Complex  $\Delta$  plane for  $B_s$  mixing. The green shaded area corresponds roughly to the experiment at the  $1\sigma$  level [114]. The left panel shows the possible phase if one omits the  $T$  parameter, the right panel shows the impact of  $T$  on the allowed phase  $\Phi_s$ .

that too large quark masses struggle to resolve the tension in the flavor sector should future experiments confirm e.g. a phase of the order of  $30^\circ$ . In Fig.6 we show the distribution of the scatter plot for “light” ( $< 440$  GeV) and “heavy” ( $> 440$  GeV)  $t'$  masses<sup>12</sup>.

As a final remark, we would like to point out that the naively expected huge contributions to  $\Delta_{B_s}$  due to the heavy  $t'$  can (still) be veiled by the mechanism described in [50]. The new contributions in the SM4 fall into two classes. The first class contains the ‘direct’ effects of the new heavy fermions, i.e., the contributions of diagrams with at least one  $t'$  in the loop. The second class is more subtle as it includes the ‘indirect’ effects of the SM4 scenario. These contributions arise from the breaking of the  $3 \times 3$  unitarity; the standard model CKM elements have to be ‘thinned’ out to accomodate for the non-zero values of the fourth row and column matrix elements: this results in a modification of the SM-like contributions.

These two sets of contributions are both sizeable (typically large enough to violate at least one experimental bound), but they can cancel to a very large extent. Here is one example for the case of  $\Delta_{B_s}$  which survived all our constraints:

$\theta_{12}$	$\theta_{13}$	$\theta_{23}$	$\theta_{14}$	$\theta_{24}$	$\theta_{34}$	
0.2275	0.003409	0.04036	0.01701	0.08392	0.1457	
$\delta_{13}$	$\delta_{14}$	$\delta_{24}$	$m_{t'}$	$m_{b'}$	$m_{l_4}$	$m_{\nu_4}$
1.019	0.91825	1.0787	385 GeV	378 GeV	602 GeV	636 GeV

This parameter set yields

$$\Delta_{B_s} = 1 + \underbrace{0.5379 - 0.9016i}_{\text{direct}} \underbrace{-0.4196 + 0.4262i}_{\text{indirect}} = 1.2 e^{-i23^\circ}. \quad (3.79)$$

One sees that each contribution is of the size of the Standard model result; however, direct and indirect contribution have opposite signs and cancel to a large extent. Note that in this case the 3-4 mixing is strong enough to allow for almost degenerate quark masses.

<sup>12</sup>Note that the points corresponding to “heavy”  $t'$  quarks are placed on top of the one corresponding to “light”  $t'$ s. A light  $t'$  and a simultaneous small phase  $\Phi_s$  are, of course, still possible.



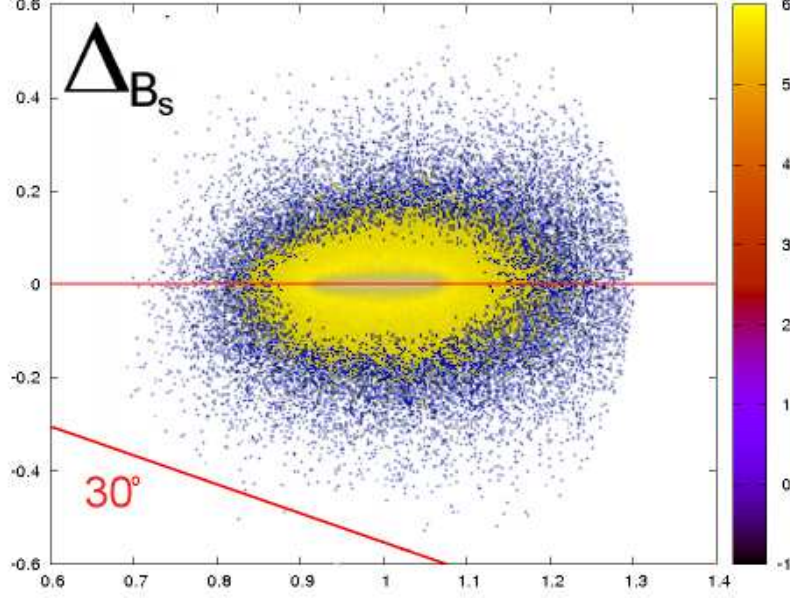


Figure 6: Dependence of  $\Delta_{B_s}$  on the  $t'$  mass. The dark blue points correspond to masses below 440 GeV, the yellow (light grey) point correspond to masses heavier than 440 GeV. The red line indicates a phase  $\Phi_s$  of  $30^\circ$ .

## 4 Taylor expansion of $V_{CKM4}$

In [50] we gave a Wolfenstein-like expansion of the CKM matrix for four fermion generations, which allows for a first estimate of possible effects of the fourth generation. Since the inclusion of the electroweak oblique parameters led to tighter constraints for the mixing, one can now further improve this expansion.

In the SM3 the hierarchy of the mixing between the three quark families can conveniently be visualized via a Taylor expansion in the small CKM element  $V_{us} \approx 0.2255 = \lambda$ : the Wolfenstein parameterization [129].

Following [130]<sup>13</sup> we define

$$V_{ub} = s_{13}e^{-i\delta_{13}} =: A\lambda^4(\tilde{\rho} + i\tilde{\eta}), \quad (4.1)$$

$$V_{us} = s_{12}(1 + \mathcal{O}(\lambda^8)) =: \lambda, \quad (4.2)$$

$$V_{cb} = s_{23}(1 + \mathcal{O}(\lambda^8)) =: A\lambda^2. \quad (4.3)$$

For the case of 4 generations one also needs the possible size, i.e. the power in  $\lambda$  of the new CKM matrix elements. Using the results of the previous section we find:

$$|V_{ub'}| \leq 0.0535 \approx 1.05\lambda^2 \quad (1.05\lambda^2),$$

$$|V_{cb'}| \leq 0.123 \approx 0.54\lambda^1 \approx 2.38\lambda^2 \quad (2.8\lambda^2),$$

$$|V_{tb'}| \leq 0.35 \approx 1.55\lambda^1 \quad (3.0\lambda^1).$$

<sup>13</sup>Note that due to historical reasons the element  $V_{ub}$  is typically defined to be of order  $\lambda^3$ , while it turned out that it is numerically of order  $\lambda^4$ .

In brackets we show the results of our previous analysis in [50]. The biggest effect of the inclusion of the electroweak precision constraints was the reduction of the allowed mixing between the third and fourth family. The mixing between the first and the fourth family can still be considerably larger than the mixing between the first and the third family. This bound is still dominated by  $D$ -mixing.

- Defining the  $V_{ub'}$  as

$$V_{ub'} = s_{14} e^{-i\delta_{14}} =: \lambda^2 (x_{14} - iy_{14}) ,$$

one obtains

$$\begin{aligned} \Rightarrow s_{14} &= \lambda^2 \sqrt{x_{14}^2 + y_{14}^2} , \\ \Rightarrow c_{14} &= 1 - \lambda^4 \frac{x_{14}^2 + y_{14}^2}{2} + O(\lambda^8) . \end{aligned} \quad (4.4)$$

The parameters  $x_{14}$  and  $y_{14}$  are effectively smaller than or equal to 1 for all cases.

- Let us further define the matrix element  $V_{cb'}$  via

$$V_{cb'} = c_{14} s_{24} e^{-i\delta_{24}} =: (x_{24} - iy_{24}) \lambda^1 . \quad (4.5)$$

Comparison with Eq. (2.3) then gives

$$\begin{aligned} \Rightarrow s_{24} e^{-i\delta_{24}} &= (x_{24} - iy_{24}) \lambda + O(\lambda^5) , \\ \Rightarrow c_{24} &= 1 + \frac{1}{2} (-x_{24}^2 - y_{24}^2) \lambda^2 + O(\lambda^5) . \end{aligned} \quad (4.6)$$

- the last ingredient is the element  $V_{tb'}$ :

$$V_{tb'} = c_{14} c_{24} s_{34} =: B\lambda \quad (4.7)$$

and therefore

$$\begin{aligned} \sin(\theta_{34}) &= B\lambda + \frac{1}{2} \lambda^3 (Bx_{24}^2 + By_{24}^2) + \mathcal{O}(\lambda^5) , \\ \cos(\theta_{34}) &= 1 - \frac{B^2 \lambda^2}{2} + \frac{1}{8} \lambda^4 (-B^4 - 4B^2 x_{24}^2 - 4B^2 y_{24}^2) + \mathcal{O}(\lambda^5) . \end{aligned} \quad (4.8)$$

Expanding the CKM4 matrix up to and including order  $\lambda^4$ , the matrix elements take the form :

$$\begin{aligned} V_{ud} &= 1 - \frac{\lambda^2}{2} + \frac{1}{8} \lambda^4 (-4x_{14}^2 - 4y_{14}^2 - 1) + O(\lambda^5) , & V_{us} &= \lambda , \\ V_{ub} &= A(\tilde{\rho} - i\tilde{\eta}) \lambda^4 , & V_{ub'} &= (x_{14} - iy_{14}) \lambda^2 , \end{aligned} \quad (4.9)$$

$$\begin{aligned}
V_{cd} &= -\lambda + \frac{1}{2}\lambda^3 (x_{24}^2 + y_{24}^2) - (x_{14} + iy_{14})(x_{24} - iy_{24})\lambda^4 + O(\lambda^5) , \\
V_{cs} &= 1 + \frac{1}{2}\lambda^2 (-x_{24}^2 - y_{24}^2 - 1) + \\
&\quad + \frac{1}{8}\lambda^4 (-4A^2 - 2x_{24}^2 (y_{24}^2 - 1) - x_{24}^4 - y_{24}^4 + 2y_{24}^2 - 1) + O(\lambda^5) , \\
V_{cb} &= A\lambda^2 , \\
V_{cb'} &= \lambda^2 (x_{24} - iy_{24}) , 
\end{aligned} \tag{4.10}$$

$$\begin{aligned}
V_{td} &= \lambda^3 (A - Bx_{14} - iBy_{14}) + (-iA\eta - A\rho + Bx_{24} + iBy_{24})\lambda^4 + O(\lambda^5) , \\
V_{ts} &= -A\lambda^2 - B\lambda^3 (x_{24} + iy_{24}) + \frac{1}{2}\lambda^4 (AB^2 - Ax_{24}^2 - Ay_{24}^2 + A - 2Bx_{14} - 2iBy_{14}) + O(\lambda^5) , \\
V_{tb} &= 1 - \frac{B^2\lambda^2}{2} + \frac{1}{8}\lambda^4 (-4A^2 - B^4 - 4B^2x_{24}^2 - 4B^2y_{24}^2) + O(\lambda^5) , \\
V_{tb'} &= B\lambda , 
\end{aligned} \tag{4.11}$$

$$\begin{aligned}
V_{t'd} &= \lambda^2 (-x_{14} - iy_{14}) + \lambda^3 (x_{24} + iy_{24}) + \\
&\quad + \frac{1}{2}\lambda^4 (-2AB + (x_{14} + iy_{14})(B^2 + x_{24}^2 + y_{24}^2) + x_{14} + iy_{14}) + O(\lambda^5) , \\
V_{t's} &= \lambda^2 (-x_{24} - iy_{24}) + \lambda^3 (AB - x_{14} - iy_{14}) + \frac{1}{2}(B^2 + 1)\lambda^4 (x_{24} + iy_{24}) + O(\lambda^5) , \\
V_{t'b} &= -B\lambda - \frac{1}{2}\lambda^3 (B(x_{24}^2 + y_{24}^2)) - A\lambda^4 (x_{24} + iy_{24}) + O(\lambda^5) , \\
V_{t'b'} &= 1 + \frac{1}{2}\lambda^2 (-B^2 - x_{24}^2 - y_{24}^2) + \\
&\quad + \frac{1}{8}\lambda^4 (-B^4 - 2B^2x_{24}^2 - 2B^2y_{24}^2 - 2x_{24}^2y_{24}^2 - x_{24}^4 - 4x_{14}^2 - y_{24}^4 - 4y_{14}^2) + O(\lambda^5) . 
\end{aligned} \tag{4.12}$$

Naturally, this expansion cannot take into account correlations among the various CKM elements. However, the expansion is quite useful if one wants a rough estimate of the maximal size of a product of CKM elements determining the impact of the fourth generation on a certain process.

## 5 Conclusion

We have investigated the experimentally allowed parameter range for a hypothetical  $4 \times 4$  quark mixing matrix. Therefore we extended our previous study [50] of the CKM like mixing



constraints of a fourth generation of quarks.

Besides the tree-level determinations of the  $3 \times 3$  CKM elements we also included the angle  $\gamma$  of the unitarity triangle, which turned out to be a rather severe bound for the phases of  $V_{CKM4}$ . Next we included the electroweak  $S$ ,  $T$ ,  $U$  parameters. Here we reproduced some of the results from [62], in particular we also excluded the three examples of very large mixing presented in [50]. In this paper we have included for the first time the full CKM dependence of the  $T$  and the  $U$  parameter. Doing so, we found that a mass degenerate fourth family of quarks is not excluded in that respect we differ e.g. from [11, 74]. While degenerate quark masses can also arise if the lepton masses are adjusted accordingly, see Sect. 2.2.3 and [14], including the full CKM dependence allows for a greater ‘flexibility’ in the parameter space: e.g. only then a simultaneous separate degeneracy of leptons and quarks of the fourth generation would not be excluded — this may be of interest if one wants to invoke a symmetry to motivate the tiny mass splittings. In addition we found also that large values of the parameter  $U$  are not excluded a priori; only after applying the  $T$  parameter constraint we are left with small values of  $U$ .

Concerning the FCNC constraints we studied  $K$ -,  $D$ -,  $B_d$ -,  $B_s$ -mixing and the decay  $b \rightarrow s\gamma$ . In contrast to [50] we also included bounds to the rare decay  $B_s \rightarrow \mu^+\mu^-$  and we improved our treatment of the QCD corrections to  $b \rightarrow s\gamma$ . It turned out that the naive bound for  $b \rightarrow s\gamma$  used in [50] was too restrictive.

Performing a scan over the whole parameter space of the SM4 we found that typically small mixing with the fourth family is favored, but still some sizeable deviations from the SM3 results are not yet excluded. We demonstrated explicitly that e.g. effects of  $\mathcal{O}(100\%)$  in  $B_s$  mixing are not excluded, yet. Concerning CP-violation in  $B_s$  mixing, we could have an almost arbitrarily large phase  $\Phi_s$  without violating the tree-level constraints. After switching on the  $T$  constraint (99 % CL of [14]) we could exclude values of the  $\mathcal{O}(-50^\circ)$  for the weak mixing phase  $\Phi_s$ , while values of  $\mathcal{O}(-20^\circ)$  can easily be obtained - this is still about two orders of magnitudes larger than the SM3 prediction  $\Phi_s = 0.24^\circ \pm 0.08^\circ$  [106]. In that respect we differ slightly from [40, 66, 67, 128]. If the real value of  $\Phi_s$  was  $-50\%$ , this could not be achieved by an extension of the SM3, which consists only of an additional fermion family<sup>14</sup>. Here new results for the  $B$ -mixing observables from TeVatron and LHCb are very desirable.

We found a minimal possible value for  $V_{tb}$  of 0.93 within the framework of the SM4, which can be compared with the result from single top production at the TeVatron [117, 118, 119]:  $V_{tb}^{\text{TeVatron}} = 0.88 \pm 0.07$ . If in nature a central value of  $V_{tb} = 0.88$  was realized, this could not be explained by the SM4 alone. Here also more precise experimental data are desirable.

In general we found a delicate interplay of electroweak and flavor observables, which strongly suggests that a separate treatment of the two sectors is not feasible.

In our opinion the next steps to determine the allowed parameter space of the SM4 consist of i) performing a combined electroweak and CKM fit, ii) including lepton mixing and iii) including even more precision observables like e.g.  $R_b$  or  $B \rightarrow K^*ll$ .

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<sup>14</sup>This statement holds only if the current allowed range for the  $S$  and  $T$  parameters will not change.

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## References

- [1] P. H. Frampton, P. Q. Hung and M. Sher, “Quarks and leptons beyond the third generation,” *Phys. Rept.* **330** (2000) 263 [arXiv:hep-ph/9903387].
- [2] B. Holdom, W. S. Hou, T. Hurth, M. L. Mangano, S. Sultansoy and G. Unel, “Four Statements about the Fourth Generation,” *PMC Phys. A* **3** (2009) 4 [arXiv:0904.4698 [hep-ph]].
- [3] B. Holdom, “Negative T?,” *Phys. Rev. D* **54** (1996) 721 [arXiv:hep-ph/9602248].
- [4] M. Maltoni, V. A. Novikov, L. B. Okun, A. N. Rozanov and M. I. Vysotsky, “Extra quark-lepton generations and precision measurements,” *Phys. Lett. B* **476** (2000) 107 [arXiv:hep-ph/9911535].
- [5] H. J. F. He, N. Polonsky and S. f. Su, “Extra families, Higgs spectrum and oblique corrections,” *Phys. Rev. D* **64** (2001) 053004 [arXiv:hep-ph/0102144].
- [6] V. A. Novikov, L. B. Okun, A. N. Rozanov and M. I. Vysotsky, “Extra generations and discrepancies of electroweak precision data,” *Phys. Lett. B* **529** (2002) 111 [arXiv:hep-ph/0111028].
- [7] V. A. Novikov, L. B. Okun, A. N. Rozanov and M. I. Vysotsky, “Mass of the Higgs versus fourth generation masses,” *JETP Lett.* **76** (2002) 127 [*Pisma Zh. Eksp. Teor. Fiz.* **76** (2002) 158] [arXiv:hep-ph/0203132].
- [8] T. Yanir, “Phenomenological constraints on extended quark sectors,” *JHEP* **0206** (2002) 044 [arXiv:hep-ph/0205073].
- [9] J. M. Frere, A. N. Rozanov and M. I. Vysotsky, “Mass and decays of Brout-Englert-Higgs scalar with extra generations,” *Phys. Atom. Nucl.* **69** (2006) 355 [arXiv:hep-ph/0412080].
- [10] J. Alwall *et al.*, “Is  $V_{tb} = 1$ ?,” *Eur. Phys. J. C* **49** (2007) 791 [arXiv:hep-ph/0607115].
- [11] G. D. Kribs, T. Plehn, M. Spannowsky and T. M. P. Tait, “Four Generations and Higgs Physics,” *Phys. Rev. D* **76** (2007) 075016 [arXiv:0706.3718 [hep-ph]].
- [12] V. A. Novikov, A. N. Rozanov and M. I. Vysotsky, “Once more on extra quark-lepton generations and precision measurements,” arXiv:0904.4570 [hep-ph].
- [13] M. I. Vysotsky, “Fourth quark-lepton generation and precision measurements,” arXiv:0910.3100 [hep-ph].
- [14] J. Erler and P. Langacker, “Precision Constraints on Extra Fermion Generations,” arXiv:1003.3211 [hep-ph].
- [15] M. Hashimoto, “Constraints on Mass Spectrum of Fourth Generation Fermions and Higgs Bosons,” arXiv:1001.4335 [hep-ph].

- [16] H. Flacher, M. Goebel, J. Haller, A. Hocker, K. Moenig and J. Stelzer, “Gfitter - Revisiting the Global Electroweak Fit of the Standard Model and Beyond,” *Eur. Phys. J. C* **60** (2009) 543 [arXiv:0811.0009 [hep-ph]].
- [17] W. S. Hou, “CP Violation and Baryogenesis from New Heavy Quarks,” *Chin. J. Phys.* **47** (2009) 134 [arXiv:0803.1234 [hep-ph]].
- [18] W. S. Hou, Y. Y. Mao and C. H. Shen, “Leading Effect of CP Violation with Four Generations,” arXiv:1003.4361 [hep-ph].
- [19] M. S. Carena, A. Megevand, M. Quiros and C. E. M. Wagner, “Electroweak baryogenesis and new TeV fermions,” *Nucl. Phys. B* **716** (2005) 319 [arXiv:hep-ph/0410352].
- [20] R. Fok and G. D. Kribs, “Four Generations, the Electroweak Phase Transition, and Supersymmetry,” *Phys. Rev. D* **78** (2008) 075023 [arXiv:0803.4207 [hep-ph]].
- [21] Y. Kikukawa, M. Kohda and J. Yasuda, “The strongly coupled fourth family and a first-order electroweak phase transition (I) quark sector,” *Prog. Theor. Phys.* **122** (2009) 401 [arXiv:0901.1962 [hep-ph]].
- [22] P. Q. Hung, “Minimal SU(5) resuscitated by long-lived quarks and leptons,” *Phys. Rev. Lett.* **80** (1998) 3000 [arXiv:hep-ph/9712338].
- [23] P. Q. Hung and C. Xiong, “Renormalization Group Fixed Point with a Fourth Generation: Higgs-induced Bound States and Condensates,” arXiv:0911.3890 [hep-ph].
- [24] P. Q. Hung and C. Xiong, “Renormalization Group Fixed Point with a Fourth Generation: Solution to the hierarchy problem,” arXiv:0911.3892 [hep-ph].
- [25] B. Holdom, “Heavy Quarks And Electroweak Symmetry Breaking,” *Phys. Rev. Lett.* **57** (1986) 2496 [Erratum-ibid. **58** (1987) 177].
- [26] J. Carpenter, R. Norton, S. Siegemund-Broka and A. Soni, “Properties of a Composite Higgs Particle in a dynamically broken Formulation of the Standard Model,” *Phys. Rev. Lett.* **65** (1990) 153.
- [27] S. F. King, “Is electroweak Symmetry broken by a fourth Family of Quarks?,” *Phys. Lett. B* **234** (1990) 108.
- [28] C. T. Hill, M. A. Luty and E. A. Paschos, “Electroweak symmetry breaking by fourth generation condensates and the neutrino spectrum,” *Phys. Rev. D* **43** (1991) 3011.
- [29] P. Q. Hung and G. Isidori, “Anatomy of the Higgs mass spectrum,” *Phys. Lett. B* **402** (1997) 122 [arXiv:hep-ph/9609518].
- [30] B. Holdom, “The discovery of the fourth family at the LHC: What if?,” *JHEP* **0608** (2006) 076 [arXiv:hep-ph/0606146].
- [31] G. Burdman, L. Da Rold, O. Eboli and R. Matheus, “A Strongly Coupled Fourth Generation at the LHC,” *Phys. Rev. D* **79** (2009) 075026 [arXiv:0812.0368 [hep-ph]].
- [32] M. Hashimoto and V. A. Miransky, “Dynamical electroweak symmetry breaking with superheavy quarks and 2+1 composite Higgs model,” arXiv:0912.4453 [hep-ph].

- [33] O. Antipin, M. Heikinheimo and K. Tuominen, “The Next Generation,” arXiv:1002.1872 [hep-ph].
- [34] D. Delepine, M. Napsuciale and C. A. Vaquera-Araujo, “Dynamical symmetry breaking with a fourth generation,” arXiv:1003.3267 [hep-ph].
- [35] Yu. A. Simonov, “Coherent mixing in three and four quark generations,” arXiv:1004.2672 [hep-ph].
- [36] G. Burdman and L. Da Rold, “Electroweak Symmetry Breaking from a Holographic Fourth Generation,” JHEP **0712** (2007) 086 [arXiv:0710.0623 [hep-ph]].
- [37] G. Burdman, L. Da Rold and R. D. Matheus, “The Lepton Sector of a Fourth Generation,” arXiv:0912.5219 [hep-ph].
- [38] A. Soni, “The ‘4th generation’, B-CP anomalies & the LHC,” arXiv:0907.2057 [hep-ph].
- [39] A. Soni, A. K. Alok, A. Giri, R. Mohanta and S. Nandi, “The fourth family: a natural explanation for the observed pattern of anomalies in B-CP asymmetries,” Phys. Lett. B **683** (2010) 302 [arXiv:0807.1971 [hep-ph]].
- [40] W. S. Hou, M. Nagashima and A. Soddu, “Large Time-dependent CP Violation in  $B_s^0$  System and Finite  $D^0 - \bar{D}^0$  Mass Difference in Four Generation Standard Model,” Phys. Rev. D **76** (2007) 016004 [arXiv:hep-ph/0610385].
- [41] A. Arhrib and W. S. Hou, “Flavor changing neutral currents involving heavy quarks with four generations,” JHEP **0607** (2006) 009 [arXiv:hep-ph/0602035].
- [42] W. S. Hou, M. Nagashima and A. Soddu, “Difference in  $B^+$  and  $B^0$  direct CP asymmetry as effect of a fourth generation,” Phys. Rev. Lett. **95** (2005) 141601 [arXiv:hep-ph/0503072].
- [43] W. S. Hou, R. S. Willey and A. Soni, “Implications of a Heavy Top Quark and a Fourth Generation on the Decays  $B \rightarrow K$  Lepton+ Lepton-,  $K$  Neutrino anti-neutrino,” Phys. Rev. Lett. **58** (1987) 1608 [Erratum-ibid. **60** (1988) 2337].
- [44] W. S. Hou, A. Soni and H. Steger, “Charmless Decays Of Bottom Mesons And A Fourth Generation,” Phys. Rev. Lett. **59** (1987) 1521.
- [45] H. Lacker and A. Menzel, “Simultaneous Extraction of the Fermi constant and PMNS matrix elements in the presence of a fourth generation,” arXiv:1003.4532 [hep-ph].
- [46] A. B. Bracic and N. S. Mankoc Borstnik, “On the origin of families of fermions and their mass matrices,” Phys. Rev. D **74** (2006) 073013 [arXiv:hep-ph/0512062].
- [47] D. Bennett *et al.*, “Proceedings to the 10th Workshop ‘What Comes Beyond the Standard Models’, arXiv:0711.4681 [hep-ph].
- [48] K. Belotsky, M. Khlopov and K. Shibaev, “Stable quarks of the 4th family?,” arXiv:0806.1067 [astro-ph].
- [49] N. S. M. Borstnik, “The ‘approach unifying spin and charges’ predicts the fourth family and a stable family forming the dark matter clusters,” arXiv:1005.2288 [hep-ph].

- [50] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, “How Much Space Is Left For A New Family Of Fermions?,” *Phys. Rev. D* **79** (2009) 113006 [arXiv:0902.4883 [hep-ph]].
- [51] E. Arik *et al.*, “Search for the fourth family up quarks at CERN LHC,” *Phys. Rev. D* **58** (1998) 117701.
- [52] W. Skiba and D. Tucker-Smith, “Using jet mass to discover vector quarks at the LHC,” *Phys. Rev. D* **75** (2007) 115010 [arXiv:hep-ph/0701247].
- [53] B. Holdom, “ $t'$  at the LHC: The physics of discovery,” *JHEP* **0703** (2007) 063 [arXiv:hep-ph/0702037].
- [54] B. Holdom, “The heavy quark search at the LHC,” *JHEP* **0708** (2007) 069 [arXiv:0705.1736 [hep-ph]].
- [55] V. E. Ozcan, S. Sultansoy and G. Unel, “Search for 4th family quarks with the ATLAS detector,” arXiv:0802.2621 [hep-ex].
- [56] J. A. Aguilar-Saavedra, “Identifying top partners at LHC,” *JHEP* **0911** (2009) 030 [arXiv:0907.3155 [hep-ph]].
- [57] B. Holdom and Q. S. Yan, “The  $b'$  search at the LHC,” arXiv:1004.3031 [hep-ph].
- [58] V. D. Barger, E. W. N. Glover, K. Hikasa, W. Y. Keung, M. G. Olsson, C. J. Suchyta and X. R. Tata, “Higgs Boson  $Z^0$  associated production from fourth generation quarks at Super Collider energies”, *Phys. Rev. Lett.* **57**, 1672 (1986).
- [59] K. F. Chen, “Searches for fourth generation particles and heavy neutrinos at CMS experiment,” *J. Phys. Conf. Ser.* **171**, 012080 (2009).
- [60] W. Bernreuther, P. Gonzalez and M. Wiebusch, “Pseudoscalar Higgs Bosons at the LHC: Production and Decays into Electroweak Gauge Bosons Revisited,” arXiv:1003.5585 [hep-ph].
- [61] C. Liu and S. Yang, arXiv:1005.1362 [hep-ph].
- [62] M. S. Chanowitz, “Bounding CKM Mixing with a Fourth Family,” *Phys. Rev. D* **79** (2009) 113008 [arXiv:0904.3570 [hep-ph]].
- [63] M. E. Peskin and T. Takeuchi, “A New constraint on a strongly interacting Higgs sector,” *Phys. Rev. Lett.* **65** (1990) 964.  
M. E. Peskin and T. Takeuchi, “Estimation of oblique electroweak corrections,” *Phys. Rev. D* **46** (1992) 381.
- [64] V. Bashiry, N. Shirkhanghah and K. Zeynali, “CP violation, single lepton polarization asymmetry, and polarized CP asymmetry in  $B \rightarrow K^* l + l^-$  decay in the four-generation standard model,” *Phys. Rev. D* **80** (2009) 015016.
- [65] G. Eilam, B. Melic and J. Trampetic, “CP violation and the 4th generation,” *Phys. Rev. D* **80** (2009) 116003 [arXiv:0909.3227 [hep-ph]].
- [66] A. Soni, A. K. Alok, A. Giri, R. Mohanta and S. Nandi, “SM with four generations: Selected implications for rare B and K decays,” arXiv:1002.0595 [hep-ph].

- [67] A. J. Buras, B. Duling, T. Feldmann, T. Heidsieck, C. Promberger and S. Recksiegel, “Patterns of Flavor Violation in the Presence of a Fourth Generation of Quarks and Leptons,” arXiv:1002.2126 [hep-ph].
- [68] N. Cabibbo, “Unitary Symmetry and Leptonic Decays,” Phys. Rev. Lett. **10** (1963) 531.
- [69] M. Kobayashi and T. Maskawa, “CP Violation In The Renormalizable Theory Of Weak Interaction,” Prog. Theor. Phys. **49** (1973) 652.
- [70] F. J. Botella and L. L. Chau, “Anticipating The Higher Generations Of Quarks From Rephasing Invariance Of The Mixing Matrix,” Phys. Lett. B **168** (1986) 97.
- [71] H. Fritzsch and J. Plankl, “The Mixing of Quark Flavors,” Phys. Rev. D **35** (1987) 1732.
- [72] H. Harari and M. Leurer, “Recommending a Standard Choice of Cabibbo Angles and KM Phases for Any Number of Generations,” Phys. Lett. B **181** (1986) 123.
- [73] M. Battaglia *et al.*, “The CKM matrix and the unitarity triangle,” arXiv:hep-ph/0304132.
- [74] C. Amsler *et al.* [Particle Data Group], “Review of particle physics,” Phys. Lett. B **667** (2008) 1.
- [75] M. Gronau and D. London., “How to determine all the angles of the unitarity triangle from  $B_d^0 \rightarrow DK_s$  and  $B_s^0 \rightarrow D^0$ ,” Phys. Lett. B **253** (1991) 483.
- [76] M. Gronau and D. Wyler, “On determining a weak phase from CP asymmetries in charged B decays,” Phys. Lett. B **265** (1991) 172.
- [77] D. Atwood, I. Dunietz and A. Soni, “Enhanced CP violation with  $B \rightarrow KD^0\bar{D}^0$  modes and extraction of the CKM angle  $\gamma$ ,” Phys. Rev. Lett. **78** (1997) 3257 [arXiv:hep-ph/9612433].
- [78] A. Giri, Y. Grossman, A. Soffer and J. Zupan, “Determining  $\gamma$  using  $B^\pm \rightarrow DK^\pm$  with multibody D decays,” Phys. Rev. D **68**, 054018 (2003) [arXiv:hep-ph/0303187].
- [79] T. Kurimoto and A. Tomita, “New physics and CP angles measurement at B factory,” Prog. Theor. Phys. **98** (1997) 967 [arXiv:hep-ph/9706222].
- [80] H. Lacker, “Constraints on CKM matrix elements within a fourth generation”, talk given at “Second Workshop on Beyond 3 Generation Standard Model — New Fermions at the Crossroads of Tevatron and LHC Search”, 14-16 January 2010, Taipei, Taiwan.
- [81] <http://ckmfitter.in2p3.fr>
- [82] A. Hocker, H. Lacker, S. Laplace and F. Le Diberder, “A new approach to a global fit of the CKM matrix,” Eur. Phys. J. C **21** (2001) 225 [arXiv:hep-ph/0104062].
- [83] J. Charles *et al.* [CKMfitter Group], “CP violation and the CKM matrix: Assessing the impact of the asymmetric  $B$  factories,” Eur. Phys. J. C **41** (2005) 1 [arXiv:hep-ph/0406184].



- [84] D. Reusser *et al.*, “Limits on cold dark matter from the Gotthard Ge experiment,” *Phys. Lett. B* **255** (1991) 143.
- [85] T. Aaltonen *et al.* [CDF Collaboration], “Search for New Particles Leading to  $Z$ +jets Final States in  $p\bar{p}$  Collisions at  $\sqrt{s} = 1.96\text{-TeV}$ ,” *Phys. Rev. D* **76** (2007) 072006 [arXiv:0706.3264 [hep-ex]].
- [86] T. Aaltonen *et al.* [CDF Collaboration], “Search for Heavy Top-like Quarks Using Lepton Plus Jets Events in 1.96-TeV  $p\bar{p}$  Collisions,” *Phys. Rev. Lett.* **100** (2008) 161803 [arXiv:0801.3877 [hep-ex]].
- [87] T. Aaltonen *et al.* [The CDF Collaboration], “Search for Fermion-Pair Decays  $Q\bar{Q} \rightarrow (tW^\mp)(\bar{t}W^\pm)$  in Same-Charge Dilepton Events,” arXiv:0912.1057 [hep-ex].
- [88] The CDF Collaboration, “Search for Heavy Top  $t' \rightarrow Wq$  in Lepton Plus Jet Events”, CDF Conference note 9446, 2008; 10110, 2010.
- [89] P. Q. Hung and M. Sher, “Experimental constraints on fourth generation quark masses,” *Phys. Rev. D* **77** (2008) 037302 [arXiv:0711.4353 [hep-ph]].
- [90] L. M. Carpenter and A. Rajaraman, “Revisiting Constraints on Fourth Generation Neutrino Masses,” arXiv:1005.0628 [hep-ph].
- [91] C. J. Flacco, D. Whiteson, T. M. P. Tait and S. Bar-Shalom, “Direct Mass Limits for Chiral Fourth-Generation Quarks in All Mixing Scenarios,” arXiv:1005.1077 [hep-ph].
- [92] M. S. Chanowitz, M. A. Furman and I. Hinchliffe, “Weak Interactions Of Ultraheavy Fermions. 2,” *Nucl. Phys. B* **153** (1979) 402.
- [93] W. J. Marciano, G. Valencia and S. Willenbrock, “Renormalization Group Improved Unitarity Bounds On The Higgs Boson And Top Quark Masses,” *Phys. Rev. D* **40**, 1725 (1989).
- [94] M. J. G. Veltman, “Limit On Mass Differences In The Weinberg Model,” *Nucl. Phys. B* **123** (1977) 89.
- [95] M. S. Chanowitz, M. A. Furman and I. Hinchliffe, “Weak Interactions Of Ultraheavy Fermions,” *Phys. Lett. B* **78** (1978) 285.
- [96] J. Alcaraz [ALEPH Collaboration and CDF Collaboration and D0 Collaboration and an], “Precision Electroweak Measurements and Constraints on the Standard Model,” arXiv:0911.2604 [hep-ex].
- [97] <http://gfitter.desy.de/>
- [98] T. Appelquist and J. Carazzone, “Infrared Singularities And Massive Fields,” *Phys. Rev. D* **11** (1975) 2856.
- [99] C. W. Chiang, A. Datta, M. Duraishamy, D. London, M. Nagashima and A. Szynekman, “New Physics in  $B_s \rightarrow J/\psi\phi$ : a General Analysis,” *JHEP* **1004** (2010) 031 [arXiv:0910.2929 [hep-ph]].

- [100] I. I. Y. Bigi and A. I. Sanda, “Notes On The Observability Of CP Violations In B Decays,” Nucl. Phys. B **193** (1981) 85.
- [101] A. Soni, “Precocious signs of new physics: Are we eight now?,” Pramana **72**, 161 (2009).
- [102] T. Inami and C. S. Lim, “Effects Of Superheavy Quarks And Leptons In Low-Energy Weak Processes  $K_L \rightarrow \mu\bar{\mu}$ ,  $K^+ \rightarrow \pi^+\nu\bar{\nu}$  and  $K^0 \leftrightarrow \bar{K}^0$ ,” Prog. Theor. Phys. **65** (1981) 297 [Erratum-ibid. **65** (1981) 1772].
- [103] A. J. Buras, M. Jamin and P. H. Weisz, “Leading and Next-to-Leading QCD Corrections to  $\epsilon$  parameter and  $B^0 - \bar{B}^0$  Mixing in the presence of a heavy Top Quark,” Nucl. Phys. B **347** (1990) 491.
- [104] S. Herrlich and U. Nierste, “Enhancement of the  $K_L - K_S$  mass difference by short distance QCD corrections beyond leading logarithms,” Nucl. Phys. B **419** (1994) 292 [arXiv:hep-ph/9310311].
- [105] S. Herrlich and U. Nierste, “The Complete  $|\Delta S| = 2$  Hamiltonian in the Next-To-Leading Order,” Nucl. Phys. B **476** (1996) 27 [arXiv:hep-ph/9604330].
- [106] A. Lenz and U. Nierste, “Theoretical update of  $B_s - \bar{B}_s$  mixing,” JHEP **0706** (2007) 072 [arXiv:hep-ph/0612167].
- [107] K. Anikeev *et al.*, “B physics at the Tevatron: Run II and beyond,” arXiv:hep-ph/0201071.
- [108] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, “Implications of  $D^0 - \bar{D}^0$  Mixing for New Physics,” Phys. Rev. D **76**, 095009 (2007) [arXiv:0705.3650 [hep-ph]].
- [109] A. J. Buras, B. Duling, T. Feldmann, T. Heidsieck, C. Promberger and S. Recksiegel, “The Impact of a 4th Generation on Mixing and CP Violation in the Charm System,” arXiv:1004.4565 [hep-ph].
- [110] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, “ $D - \bar{D}$  mixing in the framework of the HQE revisited,” arXiv:0904.3971 [hep-ph].
- [111] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, “How large can the SM contribution to CP violation in  $D^0 - \bar{D}^0$  mixing be?,” JHEP **1003** (2010) 009 [arXiv:1002.4794 [hep-ph]].
- [112] E. Barberio, R. Bernhard, S. Blyth, O. Buchmueller, G. Cavoto, P. Chang, F. Di Lodovico, H. Flaecher, T. Gershon, L. Gibbons, R. Godang, B. Golob, G. Gomez-Ceballos, R. Harr, R. Kowalewski, H. Lacker, C.-J. Lin, D. Lopes-Pegna, V. Luth, D. Pedrini, B. Petersen, M. Purohit, O. Schneider, C. Schwanda, A. J. Schwartz, J. Smith, A. Snyder, D. Tonelli, S. Tosi, K. Trabelsi, P. Urquijo, R. Van Kooten, C. Voena, C. Weiser [Heavy Flavor Averaging Group], [arXiv:hep-ex/0808.1297].
- [113] G. Buchalla, A. J. Buras and M. E. Lautenbacher, “Weak decays beyond leading logarithms,” Rev. Mod. Phys. **68** (1996) 1125 [arXiv:hep-ph/9512380].
- [114] <http://www.slac.stanford.edu/xorg/hfag/>

- [115] G. Punzi, “Flavor physics at the Tevatron,” arXiv:1001.4886 [hep-ex].
- [116] A. Lenz, “Mixing and lifetimes of  $b$ -hadrons,” arXiv:0802.0977 [hep-ph].
- [117] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **103** (2009) 092001 [arXiv:0903.0850 [hep-ex]].
- [118] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **103** (2009) 092002 [arXiv:0903.0885 [hep-ex]].
- [119] T. E. W. Group [CDF Collaboration and D0 Collaboration], arXiv:0908.2171 [hep-ex].
- [120] C. Tarantino, “Update of the Unitarity Triangle Analysis,” arXiv:0902.3431 [hep-ph].
- [121] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, “Next-to-leading order QCD corrections to the lifetime difference of  $B_s$  mesons,” Phys. Lett. B **459** (1999) 631 [arXiv:hep-ph/9808385].
- [122] M. Beneke, G. Buchalla, A. Lenz and U. Nierste, “CP asymmetry in flavor-specific B decays beyond leading logarithms,” Phys. Lett. B **576** (2003) 173 [arXiv:hep-ph/0307344].
- [123] M. Ciuchini, E. Franco, V. Lubicz, F. Mescia and C. Tarantino, “Lifetime differences and CP violation parameters of neutral B mesons at the next-to-leading order in QCD,” JHEP **0308** (2003) 031 [arXiv:hep-ph/0308029].
- [124] CDF Public Note 9458 ;  
T. Aaltonen *et al.* [CDF Collaboration], “First Flavor-Tagged Determination of Bounds on Mixing-Induced CP Violation in  $B_s \rightarrow J/\psi\phi$  Decays,” Phys. Rev. Lett. **100** (2008) 161802 [arXiv:0712.2397 [hep-ex]];  
T. Aaltonen *et al.* [CDF collaboration], “Measurement of Lifetime and Decay-Width Difference in  $B_s^0 \rightarrow J/\psi\phi$  Decays,” Phys. Rev. Lett. **100** (2008) 121803 [arXiv:0712.2348 [hep-ex]];  
D0 Conference Note 5730-CONF;  
D0 Conference Note 5618-CONF;  
V. M. Abazov *et al.* [D0 Collaboration], “Combined D0 measurements constraining the CP-violating phase and width difference in the  $B_s^0$  system,” Phys. Rev. D **76** (2007) 057101 [arXiv:hep-ex/0702030];  
V. M. Abazov *et al.* [D0 Collaboration], “Lifetime difference and CP-violating phase in the  $B_s^0$  system,” Phys. Rev. Lett. **98** (2007) 121801 [arXiv:hep-ex/0701012].
- [125] O. Deschamps, “CKM global fit and constraints on New Physics in the B meson mixing,” arXiv:0810.3139 [hep-ph].
- [126] J. Charles, Nuclear Physics B (Proc. Suppl.) **185** (2008) 17-21.
- [127] V. M. Abazov *et al.* [The D0 Collaboration], “Evidence for an anomalous like-sign dimuon charge asymmetry,” arXiv:1005.2757 [hep-ex].
- [128] W. S. Hou and C. Y. Ma, “Flavor and CP Violation with Fourth Generations Revisited,” arXiv:1004.2186 [hep-ph].

- [129] L. Wolfenstein, “Parametrization Of The Kobayashi-Maskawa Matrix,” Phys. Rev. Lett. **51**, 1945 (1983).
- [130] A. J. Buras, M. E. Lautenbacher and G. Ostermaier, “Waiting For The Top Quark Mass,  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ ,  $B_s^0 - \bar{B}_s^0$  Mixing And CP Asymmetries In B Decays,” Phys. Rev. D **50**, 3433 (1994) [arXiv:hep-ph/9403384].